ENLIGHTENMENT TO PERFECTION

# UNIVERSITY OF NORTH BENGAL

Syllabus and Examination Scheme for

M. A.

in

# MATHEMATICS

Under **CBCS**

(To be implemented from Session 2018-19)

**Signature of Chairperson**
**DDE Expert Committee, Mathematics,**
**NBU**

**Signature of HOD**
**Department of Mathematics,**
**NBU**

# Proposed PG Syllabus in Mathematics, DDE, NBU

## Credit distribution

| Course type | Total papers | Credits | Marks |
|---|---|---|---|
| 1.Core courses(**CORE**) | 7 | $7 \times 4 = 28$ | $7 \times 75 = 525$ |
| 2.Soft core courses(**SCORE**) | 4 | $3 \times 3 + 4 = 13$ | $4 \times 75 = 300$ |
| 3. Elective courses(**ELEC**) | 4 | $4 \times 3 = 12$ | $4 \times 75 = 300$ |
| 4. Open elective courses(**OLEC**) | 1 | $1 \times 3 = 3$ | $1 \times 75 = 75$ |
| 5. Assignments | 4 | $4 \times 2 = 8$ | $4 \times 4 \times (15 + 10) = 400$ |
| Total | 16 | 64 | 1600 |

## Semester 1

| Subject course No | Course | Credit |
|---|---|---|
| DEMATH1CORE1 | Abstract algebra | 4 |
| DEMATH1CORE2 | Complex Analysis I | 4 |
| DEMATH1SCORE3 | Analysis of several variables | 3 |
| DEMATH1ELEC4 **/** DEMATH1ELEC5 | Differential Geometry **/**p-adic Analysis | 3 |
| DEMATHASSG1 | Assignment | 2 |

## Semester 2

| Subject course No | Course | Credit |
|---|---|---|
| DEMATH2CORE1 | Real Analysis | 4 |
| DEMATH2CORE2 | Point set Topology | 4 |
| DEMATH2SCORE3 | Ordinary Differential Equations | 3 |
| DEMATH2ELEC4 **/** DEMATH2ELEC5 | Theory of Rings and Modules **/** Complex Analysis II | 3 |
| DEMATHASSG2 | Assignment | 2 |

## Semester 3

| Subject course No | Course | Credit |
|---|---|---|
| DEMATH3CORE1 | Linear Algebra | 4 |
| DEMATH3CORE2 | Functional Analysis | 4 |
| DEMATH3SCORE3 | Partial Differential Equations | 3 |
| DEMATH3OLEC4 **/** DEMATH3OLEC5 | Discrete Mathematics **/** History of Mathematics | 3 |
| DEMATHASSG3 | Assignment | 2 |

## Semester 4

Choose any two courses from **DEMATH4ELEC3, DEMATH4ELEC4, DEMATH4ELEC5,** and **DEMATH4ELEC6**

| Subject Course No | Course | Credit |
|---|---|---|
| DEMATH4CORE1 | Abstract Measure theory | 4 |
| DEMATH4SCORE2 | Elementary Number theory | 4 |
| DEMATH4ELEC3 | Integral equation and Integral transform | 3 |
| DEMATH4ELEC4 | Field extension and Galois theory | 3 |
| DEMATH4ELEC5 | Algebraic Topology | 3 |
| DEMATH4ELEC6 | General theory of Integration | 3 |
| DEMATHASSG4 | Assignment | 2 |

- **Question Pattern for all Papers:** Group-A: (3 Questions out of 5)×10=30, Group-B: (5 harder problems out of 7) ×6= 30 and Group-C: (5 moderate problems) ×3=15.

# Detailed Syllabus of PG Semester-I

## Paper: DEMATH1CORE1                              4 Credits
## Abstract Algebra

Homomorphism of Groups, Isomorphism Theorems, Cayley's Theorem, Generalized Cayley's Theorem, Group Action, Conjugacy Relation, Class Equation, Cauchy's Theorem, Sylow's Theorems and applications.

Ring Homomorphism. Isomorphism Theorems, Ideals and Quotient Ring. Prime and irreducible elements. Maximal and Prime Ideals. Quotient Field of an Integral Domain. Prime Fields. Irreducible and Prime Elements in a Ring. Factorisation Domain, Unique Factorisation Domain, Principal Ideal Domain, Euclidean Domain, Ring of Polynomials.

## *References*

1. David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.
2. Joseph R. Gallian, Contemporary Abstract Algebra, Narosa Publishing House.
3. John B. Fraleigh, A First Course in Abstract Algebra, Narosa Publishing House.
4. Michael Artin, Algebra, Prentice Hall.
5. Thomas Hungerford, Algebra, Springer GTM.
6. I.N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.
7. D. S. Malik, J. N. Modrdeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.
8. J.J. Rotman, The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.

# Paper: DEMATH1CORE2                                4 Credits
# Complex Analysis I

Function of a complex variable, concept of limit and continuity, Stereographic Projection, Sequences and series of functions, Analytic function and Power series, Power series as analytic function, exponential, trigonometric, logarithmic, inverse functions, Complex integration., Cauchy-Goursat Theorem (for convex region), Winding number or index of a curve, Cauchy's integral formula, Higher order derivatives, Morera's Theorem, Cauchy's inequality and Liouville's theorem, Doubly periodic entire function, The fundamental theorem of algebra, Zeros of analytic functions, Maximum modulus principle, Hadamard's three circle theorem, Taylor's theorem, Schwarz lemma, Laurent's series, Isolated singularities, Casoratti-weierstrass theorem.

Residues. Cauchy's residue theorem, Evaluation of integrals, Rouche's theorem, Meromorphic functions, The argument principle, inverse function theorem, Branches of many valued functions with special reference to arg z , log z and $z^a$., Riemann surfaces.

Bilinear transformations, their properties and classifications, Definitions and examples of Conformal mappings, Cross-Ratio, Principle of Symmetry.

Analytic continuation, Uniqueness of direct analytic continuation, Monodromy theorem, Analytic continuation via Reflection, Uniqueness of analytic continuation along a curve, Power series method of analytic continuation.

## References

1. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990.
2. J. B. Conway, Functions of one Complex variable. Springer-Verlag. International Student Edition, Narosa Pub. House. 1980.
3. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis. Jones and Bartlett Pub. International London, 1996.
4. L. V. Ahlfors. Complex Analysis, McGraw-Hill.
5. S. Lang. Complex Analysis, Addison Wesley. 1970.
6. D. Sarason, Complex Function Theory , Hindustan Book Agency, Delhi, 1994.
7. E. Hille, Analytic Function Theory (2 vols) , Gonn & Co, 1959.
8. W.H.J. Fuchs, Topics in the Theory of Functions of one complex variable, D. Van Nostrand Co. , 1967.
9. C. Caratheodory. Theolry of ;Functions (2 vols) Chelsea Publishing Company, 1964.
10. M. Heins, Complex Function Theory. Academic Press, 1968.

11. Walter Rudin, Real and Complex Analysis, McGraw - Hill Book Co, 1966.
12. S. Saks and A. Zygmund, Analytic Functions, Monografie Matematyczne, 1952.
13. E. C. Titchmarsh, The Theory of Functions, Oxford Univ. Press, London.
14. W. A. Veech, A Second Course in Complex Aanlysis. W. A. Benjamin, 1967.
15. S. Ponnusamy, Foundations of Complex Analysis, Narosa Pub. House, 1997.

**Paper: DEMATH1SCORE3**                                                 **3 Credits**
# Analysis of Several Variables

Topology of $\mathbb{R}^n$ , $GL_n(\mathbb{R})$ etc. . Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear map. Determinant as mapping; its continuity and differentiability. Existence and meaningfulness of $e^A$ and its continuity as well as differentiability (A is a real square matrix). Higher derivatives, Chain Rule, mean value theorem for differentiable functions, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier, Sard's theorem. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustration with plenty of examples. Inverse and implicit function theorems. Picard's Theorem.

Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$. Line integrals, Surfaces in $\mathbb{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green's theorem, Gauss (Divergence) theorem and Stoke's theorem.

## *References*

1. M. Spivak: Calculus on manifolds, Benjamin (1965).
2. W. Rudin: Principles of mathematical analysis, Mc Graw-Hill.
3. T. Apostol: Mathematical Analysis
4. Munkres, J., Analysis on Manifolds.
5. T. Apostol: Calculus (Vol 2), John Wiley.

# Paper: DEMATH1ELEC4                                    3 Credits
## Differential Geometry

Curvilinear Coordinates, Elementary arc length, Length of a vector, Angle between two non-null vectors, Reciprocal Base system, Intrinsic Differentiation, Parallel vector fields.
Geometry of space curves: Serret-Frenet formulae, Equation of Straight lines, Helix, Bertrand curve.

Quick recap of multivariate calculus, Inverse Function Theorem and Implicit Function Theorem.

Regular surfaces, differential functions on surfaces, the tangent plane and the differential maps between regular surfaces, the first fundamental form, normal fields and orientability.

Gauss map, shape operator, the second fundamental form, normal and principle curvatures, Gaussian and mean curvatures.

Geodesic, Exponential map, Parallel transport, Theorem of Egregium.

Geodesic curvature, Gauss-Bonnet Theorem for simple closed curve.

## *References:*

1. Elementary Differential Geometry, Andrew Pressley, Springer, 2010.
2. Elementary Differential Geometry, Barrett O'Neill, Elsevier, 2006.
3. Elementary Differential Geometry, Christian Bär, Cambridge University Press, 2011.
4. Differential Geometry of Curves and Surfaces, Manfredo P. Do Carmo, Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458, 1976.
5. A Text Book of Differential Geometry, U. C. De, Asian Books Pvt. Ltd, 2014.
6. An Introduction to Differential Geometry (with the use of tensor Calculus), Princeton University Press, 1940.

# Paper: DEMATH1ELEC5                                    3 Credits
## P-adic Analysis

     I.     Congruences and modular equations

    II.     The p-adic norm and the p-adic numbers

III.    Some elementary p-adic analysis

IV.    The topology of Qp

V.    p-adic algebraic number theory

### *References:*

1. G. Bachman, Introduction to p-adic numbers and valuation theory, Academic Press (1964).
2. J. W. S. Cassels, Local fields, Cambridge University Press (1986).
3. F. Q. Gouv^ea, p-adic Numbers: An Introduction, 2nd edition, Springer-Verlag (1997).
4. S. Katok, p-adic analysis compared with real, American Mathematical Society (2007).
5. N. Koblitz, p-adic numbers, p-adic analysis and zeta functions, second edition, Springer-Verlag (1984).
6. S. Lang, Algebra, revised third edition, Springer-Verlag (2002).
7. K. Mahler, Introduction to p-adic numbers and their functions, second edition, Cambridge University Press (1981).
8. AM. Robert, A course in p-adic analysis, Springer-Verlag, 2000.

# Detailed Syllabus of PG Semester-II

**Paper: DEMATH2CORE1**                                     **4 Credits**

## Real Analysis

Extended real numbers, Algebraic operations and convergence in extended real number systems. Lebesgue outer measure, Measurable sets , regularity, Measurable Functions, Borel and Lebesgue measurability.

### *References:*

1. Fundamentals of Real Analysis, S K. Berberian, Springer.
2. Measure Theory and Integration, G. De Barra, New Age International Publ.
3. Real Analysis, H. L. Royden.
4. Principles of Mathematical Analysis, W. Rudin.
5. Lectures on Real Analysis, J. Yeh, World Sci.
6. *R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966*

**Paper: DEMATH2CORE2**                                     **4 Credits**

## Point-Set Topology

Axiom of choice and existence of choice function. Partially ordered set, linearly ordered set, well ordered set and product of the same kinds, Zorn's lemma, well ordering principle with special emphasis on Ordinal and Cardinal numbers.

Topological spaces, open and closed sets, basis and sub-basis, closure, interior and boundary of a set. Subspace topology. Continuous maps: properties and constructions; Pasting Lemma. Open and closed maps, Homeomorphisms. Product topology, Quotient topology and examples of Topological Manifolds. Countability and separation axioms: Urysohn's lemma, Tietze extension theorem and applications. Urysohn embedding lemma and metrization theorem for second countable spaces. Connected, path-connected and locally connected spaces. Lindelof and Compact spaces. Net, Filters

Different kinds of compactness and their identity in metric spaces. Local compactness, Compactifications, Paracompactness. Covering Spaces and Uniform Spaces.

### References:

1. J. R. Munkres, Topology: a first course, Prentice-Hall (1975).
2. G.F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw-Hill (1963).
3. M.A. Armstrong, Basic Topology, Springer.
4. J. L. Kelley, General Topology, Springer-Verlag (1975).
5. J. Dugundji, Topology, UBS (1999).
6. Stephen Willard, General Topology, Dover (2004).
7. I. P. Natanson, Theory of functions of a real variable, Vol. II. (especially for Ordinal numbers)

**Paper: DEMATH2SCORE3**                                              **3 Credits**
# Ordinary Differential Equations

Review of solution methods for first order as well as second order equations, Power Series methods with properties of Bessel functions and Legendré polynomials.

Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, continuous dependence.

Higher Order Linear Equations and linear Systems: fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Boundary Value Problems for Second Order Equations: Green's function, Sturm comparisontheorems and oscillations, eigenvalue problems.

### References

1. S. L. Ross, Differential Equations, 3rd Edn., Wiley India, 1984.
2. G.F. Simmons, Differential Equations with Applications and Historical Notes, Tata-McGrawHill 2003.
3. M.Brown, Differential Equations and Their Applications, Springer 1983.
4. W. Boyce and R. Diprima, Elementary Differential Equations and Boundary Value Problems.
5. G. Birhoff & G.C. Rofa Ordinary Differential Equations, Wily ,1978

# Paper: DEMATH2ELEC4                                               3 Credits
# Theory of Rings and Modules

**Ring Theory-** Noetherian and Artinian Rings, Hilbert Basis Theorem, Cohen's Theorem. Radicals of Rings and Modules, Primary Decomposition of Noetherian rings.

**Module theory**- Modules, sub modules, quotient modules; homomorphism and isomorphism theorems. Commutativity of Diagrams, Exact Sequences, Four Lemma, Five Lemma. Direct Sum and product of modules, free modules, cyclic modules, simple and semi-simple modules, projective and injective modules, flat modules. Fundamental Structure Theorem for finitely generated modules over a PID and its applications to finitely generated abelian groups. Embedding of a module in an injective module, Tensor product of modules, chain conditions on modules. Noetherian and Artinian modules.

## *References:*

1. Lang, S., Algebra, Addison-Wesley, 1993.
2. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag
3. Algebra, by Michael Artin, Prentice Hall.
4. Herstein, I.N., Topics in Abstract Algebra, Wiley Eastern Limited.
5. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.
6. David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.
7. T. S. Blyth, Module Theory: An Approach to Linear Algebra, Oxford University Press, 1977.
8. M. Atiyah, I.G. MacDonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.

9. Thomas Hungerford, Algebra, Springer GTM.

# Paper: DEMATH2ELEC5                                               3 Credits
# Complex Analysis II

**Harmonic Function:** Definition, Relation between Harmonic function and an analytic function, Examples, Harmonic Conjugate of a Harmonic function, Poisson's integral formula, Mean value

property, The maximum and minimum principles, Dirichlet's problem for a disc and uniqueness of its solution, Characterization of harmonic function by mean value property.

**Infinite Product:** Definition, Necessary condition for convergence, General principle of convergence, Weierstrass inequality, Convergence of Infinite Product in terms of Corresponding series, Comparison test for Convergence, Absolute Convergence, Uniform convergence.

**Integral Function:** Factorization of Integral function, Weierstrass's Primary factor, Weierstrass factorization theorem, functions of finite order, Examples, The function n(r), Exponent of Convergence of Zeros, Canonical products, Hadamard's factorization theorem, Genus, Laguerre's Theorem, a-points of Integral function, Borel's Theorem, Picard's Theorem.

## *References:*

1. H.A Priestly, Introduction to Complex Analysis, Clarendon Press Oxford, 1990.
2. J.B. Conway, Functions of one Complex variable, Springer-Verlag, International student-edition, Narosa Pub. House, 1980.
3. Liang-Shin Hahn and Bernard Epstein, Classical Complex Analysis, Jones and
4. Bartlett Pub. International London, 1996.
5. L.V. Ahlfors, Complex Analysis, McGraw.
6. S. Lang, Complex Analysis, Addison Wesley, 1970.
7. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
8. Mark J. Ablowitz and AS. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, South Asian edn. 1998.
9. E. Hille, Analytic Function Theory (2 Vols), Gonn and Co., 1959.
10. W.H.J. Fuchs, Topics in the Theory of Functions of one complex variable, D. Van Nostrand Co., 1967.
11. C. Caratheodory, Theory of Functions (2 vols), Chelsea Publishing Company,
12. 1964.
13. M. Heins, Complex Function Theory, Academic Press, 1968.
14. Walter Rudin, Real and Complex Analysis, McGraw- Hill Book Co., 1966.
15. S. Saks and A Zygmund, Analytic Functions, Monographie Matematyczne, 1952.
16. E.C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
17. W.A Veech, A Second course in Complex Analysis, WA. Benjamin, 1967.
18. *S. Ponnusamy, Foundations of Complex Analysis,Narosa Publication House, 1997*
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# Detailed Syllabus of PG Semester-III

**Paper:** DEMATH3CORE1                                             **4 Credits**
## Linear Algebra

Linear transformations, Algebra of linear transformations, Matrix representation of linear transformations. Change of Basis.

Annihilating polynomials, diagonal forms, triangular forms, Direct Sum Decompositions, Invariant Direct sums, The Primary Decomposition Theorem.

Jordan Blocks and Jordan forms. Rational Canonical Form, Generalized Jordan form over an arbitrary field.

Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Bilinear forms, Symmetric Bilinear forms, Skew - Symmetric Bilinear forms.

### *References*

1. K. Hauffman and R. Kunz, Linear Algebra, Pearson Education (INDIA), 2003.
2. G. Strang, Linear Algebra And Its Applications, 4th Edition, Brooks/Cole, 2006.
3. S. Lang, Linear Algebra, Springer, 1989.
4. David S. Dummit and Richard M. Foote, Abstract Algebra (3e), John Wiley and Sons.
5. R. Gallian Joseph, Conte mporary Abstract Algebra, Narosa Publishing House.
6. Thomas Hungerford, Algebra, Springer GTM.
7. I.N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.
8. D.S. Malik, J.M. Mordesen, M.K. Sen, Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.

**Paper:** DEMATH3CORE2                                             **4 Credits**
## Functional Analysis

Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms. Riesz Lemma, basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples. Uniform boundedness theorem and some of its consequences. Open mapping and closed graph theorems. Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces. Reflexive spaces.

Inner product spaces. Hilbert spaces. Orthonormal sets. Bessel's inequality. Comlplete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces. Self-adjoint operators, Positive, projection, normal and unitary operators.

## *References:*

1. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
2. N. Dunford and J. T. Schwartz, Linear Operators, Part I, Interscience, New York, 1958.
3. R. E. Edwards, Functional Analysis. Holt Rinehart and Winston, New York, 1965.
4. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
5. R. B. Holmes, Geometric Functional Analysis and its Applications, Springer-Verlag 1975.
6. L. V. Kantorovich and G. P. Akilov, Functional Analysis, Pergamon Press, 1982.
7. K. Kreyszig , Introductory Functional Analysis with Applications, John Wiley & Sons New York, 1978.
8. B. K. Lahi;ri, Elements of Functional Analysis, The World Press Pvt. Ltd. Calcutta, 1994.
9. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
10. L. A. Lustenik and V. J. Sobolev, Elements of Functional Analysis, Hindustan Pub. Corpn. N.Delhi 1971.
11. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw -Hill Co. New York , 1963.
12. A. E. Taylor, Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.
13. K. Yosida, Functional Analysis, 3rd edition Springer - Verlag, New York 1971.
14. J. B. Conway, A course in functional analysis, Springer-Verlag, New York 199

# Paper: DEMATH3SCORE3         3 Credits

# Partial Differential Equations

Cauchy Problems for $1^{st}$ Order Hyperbolic Equations, Method of Characteristics etc.

Classification of Second Order Partial Differential Equations: normal forms and characteristics.

Initial and Boundary Value Problems: Lagrange-Green's identity and uniqueness by energy methods.

Laplace equation: mean value property, weak and strong maximum principle, Green's function, Poisson's formula, Dirichlet's principle, existence of solution using Perron's method (without proof).

Heat equation: initial value problem, fundamental solution, weak and strong maximum principle and uniqueness results.

Wave equation: uniqueness, D'Alembert's method, method of spherical means and Duhamel's principle.

Methods of separation of variables for heat, Laplace and wave equations.

## *References:*

1. S. L. Ross, Differential Equations, 3rd Edn., Wiley India, 1984.
2. DiBenedetto, Partial Differential Equations, Birkhaüser, 1995.
3. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, 1998.
4. I.N. Sneddon Elements of Partial Differential Equations McGrawHill 1986.
5. R. Churchil & J. Brown, Fourier Series & Boundary Value Problems.
6. R.C. McOwen , Partial Differential Equations  (Pearson Edu.) 2003.

# Paper: DEMATH3OLEC4         3 Credits
# Discrete Mathematics

**Number Theory and Cryptography:** Divisibility and Modular Arithmetic, Integer, Representations and Algorithms, Primes and Greatest**,** Common Divisors, Solving Congruences, Applications of Congruences, Cryptography.

**Counting Techniques:** The Basics of Counting, The Pigeonhole Principle Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Recurrence Relations, Generating**,** Functions, Principle of Inclusion–Exclusion, Applications of Inclusion–Exclusion. Modeling with recurrence relations with examples of Fibonacci numbers and the tower of Hanoi problem, Solving recurrence relations. Divide-and-Conquer relations with examples (no theorems).Generating functions, definition with examples, solving recurrence relations using generating functions, exponential generating functions. Difference equations.

**Order Relations and Structures:** Product Sets and Partitions, Relations, Properties of Relations, Equivalence Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Boolean Functions as Boolean Polynomials. Definition and types of relations. Representing relations using matrices and digraphs, Closures of relations, Paths in digraphs, Transitive closures, Warshall's Algorithm.

**Groups and Coding Theory:** Binary Operations Revisited, Semigroups, Products and Quotients of Semigroups, Groups, Products and Quotients of Groups, Coding of Binary Information and Error Detection, Decoding and Error Correction.

**Graph Theory :** Elements of Graph Theory, Eulerian and Hamiltonian graphs, Planar Graphs, Directed Graphs, Trees, Tree traversals, binary search trees, Permutations and Combinations, Pigeonhole  principle, principle of Inclusion and Exclusion, Derangements.

## *References:*

1. Kenneth H. Rosen - Discrete Mathematics and Its Applications, Tata Mc-Graw-Hill, 7th Edition, 2012.
2. Bernard Kolman, Robert C. Busby, Sharon Cutler Ross-Discrete Mathematical Structures-Prentice Hall, 3rd Edition, 1996.
3. Grimaldi R-Discrete and Combinatorial Mathematics. 1-Pearson, Addison Wesley, 5th Edition, 2004.
4. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill, 1986.
5. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.
6. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
7. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.
8. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2nd Ed.,
9. Clark and D. A. Holton – A First Look at Graph Theory, Allied publishers.
10. D. B. West – Introduction to Graph Theory, Pearson Education Inc.,2001, 2nd Ed.,
11. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier, 1976

12. J. P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer   Science, McGraw Hill Book Co. 1997
13. S. Witala, Discrete Mathematics - A Unified Approach, McGraw Hill Book Co.

# Paper: DEMATH3OLEC5                                                  3 Credits
# History of Mathematics

Arithmetic and Geometry, Pythagorean triples, Rational points on circles, Irrational numbers.
Greek Geometry, deductive method, conic sections.
Role of number theory, Euclid algorithm, Pell's equations
Calculus, maxima-minima and tangents, Calculus of Leibnitz, Fermat's little theorem, Fermat's last theorem.
Biographical notes on Pythagorus, Euclid, Diophantus, Leibnitz, Newton, Fermat.
Leonhard Euler, Calculus, Calculus of variations,  number theory, Biography of Euler.
John Napier, Napier's algorithm, recognition, achievements, setback and influence of Lagrange's attempts to reduce calculus to Algebra, Biographical notes of Lagrange.
Mathematical proofs, proof versus non-proofs, conjectures and mathematical induction.

### References:
1. John Stillwell, Mathematics and Its History, Springer, 2001.
2.  A decade of the Berkeley math circle, The American experience, Vol 1, AMS, 2008.
3. A Historian Looks Back, The Calculus as Algebra and selected writings, MAA, 2010.
4. Elimaor, e: The story of a number, University press, 1999
5. V. S. Varadaranjan, Euler through time: A new look at old themes, Hindustan book Agency, 2006

# Detailed Syllabus of PG Semester-IV

**Paper:** DEMATH4CORE1                                     **4 Credits**

## Abstract Measure Theory

Abstract measure spaces: σ-algebra of sets, limit of sequences of sets, Borel σ-algebra, measure on a σ-algebra, measurable space and measure space.

Borel and Lebesgue measurability of functions on **R.** cantor ternary set and Cantor-Lebesgue function. Completion of Measure Space.

### *References:*

1. Fundamentals of Real Analysis, S K. Berberian, Springer.
2. Measure Theory and Integration, G. De Barra, New Age International Publ.
3. Real Analysis, H. L. Royden.
4. Principles of Mathematical Analysis, W. Rudin.
5. Lectures on Real Analysis, J. Yeh, World Sci.
6. R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, 1966

**Paper:** DEMATH4SCORE2                                     **4 Credits**
## Elementary Number Theory

Division algorithm, Greatest common divisor, Euclidean algorithm, Diophantine equations. Fundamental Theorem of Arithmetic.

Congruences, Binary and Decimal Representations of integers, Chinese remainder theorem, Fermat's Little Theorem, Pseudoprimes, Euler's Theorem, Wilson's theorem, linear congruences, order of an integer modulo a prime, primitive roots for primes, quadratic residues, Legendre's Symbol and its properties, Law of Quadratic Reciprocity.

Arithmetic functions like Mobius function, Euler phi function, greatest integer function etc. Mobius inversion formula, Dirichlet's product of arithmetical functions, Dirichlet's inverse, The Mangoldt function, Multiplicative and Completely Multiplicative functions, Formal Power Series, The Bell series of an arithmetical function, Derivatives of an arithmetical function.

### References:

1. David M. Burton, Elementary Number Theory, University of New Hampshire.
2. G.H. Hardy, and , E.M. Wrigh,. An Introduction to the Theory of Numbers (6th ed, Oxford University Press, (2008).
3. W.W. Adams and L.J. Goldstein, Introduction to the Theory of Numbers, 3rd ed., Wiley Eastern, 1972.
4. A. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press, Cambridge, 1984.
5. I. Niven and H.S. Zuckerman, An Introduction to the Theory of Numbers, 4th Ed., Wiley, New York, 1980.
6. T.M. Apostol, Introduction to Analytic number theory, UTM, Springer, (1976).
7. J. W. S Cassel, A. Frolich, Algebraic number theory, Cambridge.
8. M Ram Murty, Problems in analytic number theory, springer.
9. M Ram Murty and Jody Esmonde, Problems in algebraic number theory, springer.

## Paper: DEMATH4ELEC3               3 Credits
# Integral Equation and Integral Transform

**Integral equations:** classifications, successive approximations, separable kernels, Fredholm alternative, Hilbert-Schemidt theory of symmetric kernels, Construction of Green's function, Convoluted Kernels, Abels equations and solutions.
Calculus of Variations, Euler-Lagrange's equations, Geodesics, Minimum surface of revolution, Isoperimetric problems, Brachistochrone problem.

**Integral transforms**: Laplace and Fourier transforms, Applications to boundary Value Problems, Mellin & Hanckels transform, Inversion formulae, Bromwich Integral, Convolutions and applications, Distributions and their transforms. Applications to Wave, Heat and Laplace equations.

### References:

1. M. Gelfand and S. V. Fomin. Calculus of Variations, Prentice Hall.
2. Linear Integral Equation: W.V. Lovitt (Dover).
3. Integral Equations, Porter and Stirling, Cambridge.
4. The Use of Integral Transform, I.n. Sneddon, Tata-McGrawHill, 1974
5. R. Churchil & J. Brown Fourier Series and Boundary Value Problems, McGraw-Hill, 1978
6. D. Powers, Boundary Value Problems Academic Press, 1979.

# Paper: DEMATH4ELEC4           3 Credits
## Field Extension and Galois Theory

Field extension – Algebraic and transcendental Extensions. Separable and Inseparable extensions. Perfect fields, Artin's Theorem, Normal extensions. Splitting fields of a polynomial. Finite fields. Primitive elements, Primitive Element Theorem, Algebraically closed fields, Algebraic closure of a field and its existence.

Galois extensions. Galois Group of automorphisms and Galois Theory, Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals. Insolvability of the general equation of degree 5 (or more) by radicals.

### *References*

1. *M. Artin, Algebra, Perentice -Hall of India, 1991.*
2. *P.M. Cohn, Algebra, vols, I,II, & III, John Wiley & Sons, 1982, 1989, 1991.*
3. *N. Jacobson, Basic Algebra, vols. I & II, W. H. Freeman, 1980 (also published by Hindustan  Publishing Company)*
4. *S. Lang. Algebra, 3rd edn. Addison-Weslley, 1993.*
5. *I.S. Luther and I.B.S. Passi, Algebra, Vol.III-Modules, Narosa Publishing House.*
6. *D. S. Malik, J. N. Modrdeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.*
7. *Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999*
8. *I. Stweart, Galois Theory, 2nd edition, Chapman and Hall, 1989.*
9. *J.P. Escofier, Galois theory, GTM Vol.204, Springer, 2001.*

# Paper: DEMATH4ELEC5           3 Credits
## Algebraic Topology

Homotopy Theory : Fundamental Groups. Fundamental groups of Circle, Sphere and some surfaces. Geometrical construction of group structure on circle (in fact on any conic), Separation Theorem in the plane, Classification of surfaces. Simplical complex, Homology, Cohomology .

### *References :*

1. Satya Deo ,Algebraic Topology-A Primer , Hindustan Book Agency
2. James r. Munkres, topology ,PHI
3. Anant  R. Shastri, Basic Algebraic Topology, CRC Press Book.

**Paper:** DEMATH4ELEC6                                                                  **3 Credits**

# General Theory of Integration

Tagged Gauge Partitions. Definitions, Cousins Theorem, Right-left Procedure, Straddle Lemma, Application in continuity, Intrinsic Power.
Henstock–Kurzweil Integral. Definition and basic properties. Fundamental Theorem, Saks-Henstock Lemma, Inclusion of the Lebesgue integral. SqueezTheorem, Vitali- Covering Theorem, Differentiation Theorem, Characterization Theorem.

## *References:*

1. A Modern Theory of Integration, R. G. Bartle, AMS
2. Theories of Integration, Douglas S. Kurtz & Charles W. Swartz, World Scientific.
3. Lanzhou Lectures on Henstock Integration, Lee Peng Yee, World Scintific.
4. The Riemann, Lebesgue and General Riemann Integrals, A.G. Das, Narosa.
5. The general Theory of integration, R. Henstock, Clarendon Press.