

**M.Sc Semester –III**

**Assignment**

**Subject- Mathematics**

**Course –Functional Analysis**

**Subject Course No.-DEMATH3CORE2**

**Total Marks-25**

**Group-A**

**Answer any one of the following questions (15 marks)**

1. (a) If  $z$  is any fixed element of an inner product space  $X$ , show that  $f(x) = \langle x, z \rangle$  defines a bounded linear functional  $f$  on  $X$ , of norm  $\|z\|$ .  
(b) Show that dual space of  $l^1$  is  $l^\infty$ .
2. State and prove the open mapping Theorem and then deduce the closed graph Theorem from it.

**Group-B**

**Answer any one of the following questions (10marks)**

1. Let  $\{x_n\}$  be a sequence in a normed space  $X$ . Then prove that
  - (i) Strong convergence implies weak convergence with same limit.
  - (ii) The converse of (i) is not generally true.
  - (iii) If  $\dim X < \infty$ , then weak convergence implies strong convergence
2. Suppose  $T: \mathcal{D}(T) \rightarrow Y$  be a bounded linear operator, where  $\mathcal{D}(T)$  lies in normed space  $X$  and  $Y$  is Banach space. Then prove that  $T$  has an extension  $\check{T}: \overline{\mathcal{D}(T)} \rightarrow Y$  where  $\check{T}$  is a bounded linear operator of the form  $\|\check{T}\| = \|T\|$ .