M.Sc Semester –III

Assignment

Subject- Mathematics

Course –Linear Algebra

Subject Course No.-DEMATH3CORE1

Total Marks-25

Group-A

Answer any one of the following questions (15 marks)

1. Define Symmetric Bilinear form. Let V be a finite dimensional vector space over a field F not of characteristic two. Then prove that every symmetric bilinear form on V is diagonalizable.
2. (a) Let V be finite dimensional vector space and $T: V \to V$ be linear and $V = R(T) + N(T)$. Then prove that $V = R(T) \oplus N(T)$. Give an example to show that above cannot be proved without assuming that V is finite-dimensional.

   (b) Prove that if $V = W \oplus W^{\perp}$ and T is the projection on W to $W^{\perp}$, then $T = T^{\star}$.

Group-B

Answer any one of the following questions (10 marks)

1. Let V be a vector space and $T: V \to V$ be a linear transformation. Prove that if the minimal polynomial of T is of the form $p(t) = (\varphi(t))^m$, then there exists a rational canonical basis for T.

2. Find the Jordan canonical form J and a Jordan canonical basis $\beta$of the linear operator T on $P_3(\mathbb{R})$ defined by $T\big(f(x)\big) = f''(x) + 2f(x), \forall\, f(x) \in P_3(\mathbb{R})$.