Group-A

Answer **any one** of the following questions (15 marks)

1. State and prove Monotone Convergence theorem for sequence of measurable sets.

2. Let μ^* is a metric outer measure on a metric space (X, d). Let $(A_n : n \in \mathbb{N})$ be an increasing sequence in $\mathcal{B}(X)$ and $A = \lim_{n\to\infty} A_n$. If A_n and $A_n \setminus A_{n+1}$ are positively separated for every $n \in \mathbb{N}$, then $\mu^*(A) = \lim_{n\to\infty} \mu^*(A_n)$.

Group-B

Answer **any one** of the following questions (10 marks)

1. Let E be the set of all irrational numbers in the interval $(0,1)$. Show that for every $\varepsilon \in (0,1)$ there exists a closed set C in $\mathbb{R}$ such that $C \subset E$ and $\mu_L(C) > 1 - \varepsilon$.

2. Let μ^* be a metric outer measure on a set X. Show that a non- μ^* mesurable subset of X exists if and only if μ^* is not countably additive on $\mathcal{B}(X)$.