Assignment

Subject-Mathematics

Semester I

Course-Analysis of Several Variables

Paper Code-DEMATH1SCORE3

Total Marks-25

<u>Group – A</u>

Answer **any one** question from the following questions (15 *marks*)

1. Suppose that f is a C^1 mapping of an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^n$, $f'(a)$ is invertible for some $a \in E$ and $b = f(a)$. Then prove that

(i) there exists open sets U and V in $\mathbb{R}^n$ such that $a \in U$, $b \in V$, f is one to one in U and $f(U) = V$.

(ii) If g is the inverse of f defined in V by $g(f(x)) = x$, $x \in U$, then $g \in C^1(V)$. [15]

2. (a) Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$ and $f(x) = (f_1(x), f_2(x), ..., f_m(x))$, for all $x \in \mathbb{R}^n$ and $f_i : \mathbb{R}^n \to \mathbb{R}$, $1 \leq i \leq m$. Then prove that $D_j f_i(a)$ exists for all $1 \leq i \leq m$, $1 \leq j \leq n$ and $f'(a) = \left(D_j f_i(a)\right)_{m\times n}$ where $D_j f_i(a)$ denotes partial derivatives of f_i at $a \in \mathbb{R}^n$ w.r.t jth variable.

(b) Prove that $\lim_{||H||\to 0} \frac{|detH|}{||H||} = 0$, where $H \in M_n(\mathbb{R})$. [$10 + 5 = 15$]

<u>Group – B</u>

Answer **any one** question from the following questions (10 *marks*)

1. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be differentiable at $a \in \mathbb{R}^n$. Then show that all directional derivatives $D_u f(a)$ exists and $D_u f(a) = Df(a)(u)$ for all $u \in \mathbb{R}^n$. Give an example to show that the converse is not true in general. [10]

2. State and prove Green's Theorem in the plane. Verify Green's theorem in the plane for $\int_\Gamma \{(1 + xy^2)dx - x^2 y dy\}$, where Γ consists of the arc of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$. [10]