Assignment

Subject-Mathematics

Semester I

Course-P-adic Analysis

Paper Code-DEMATH1ELEC5

Total Marks-25

Group – A

Answer **any one** question from the following questions (15 *marks*)

1. Prove that the ring $(\hat{R}_N, +, \times)$ where sum $+$ and product $\times$ given by $\{a_n\} + \{b_n\} = \{a_n + b_n\}$, $\{a_n\} \times \{b_n\} = \{a_n b_n\}$ is commutative if R is commutative. Moreover, there is a unique norm $\hat{N}$ on $\hat{R}_N$ which satisfies $\hat{N}(\{a\}) = N(a)$ for a constant Cauchy sequence $(a_n) = (a)$ with $a \in R$; this norm is defined by

$$\hat{N}(\{c_n\}) = \lim_{n\to\infty} N(c_n)$$

as a limit in the real numbers $\mathbb{R}$. Finally, show that $\hat{N}$ is non-Archimedean if and only if N is. [15]

2. Let α_n be a sequence in $\mathbb{Z}$. Then prove that there is a convergent subsequence of $\{\alpha_n\}$. A similar result holds for each of the closed discs $\overline{D(\beta,\ t)}$, where $t \geq 0$ is a real number. [15]

Group – B

Answer **any one** question from the following questions (10 *marks*)

1. Let $f : \mathbb{Z}_p \to \mathbb{Q}_p$ be a continuous function. Then show that there is an $\alpha_0 \in \mathbb{Z}_p$ such that $b_f = |f(\alpha_0)|_p$. [10]

2. Show that the set of p-adic integers $\mathbb{Z}_p$ is a subring of $\mathbb{Q}_P$ and every element of $\mathbb{Z}_p$ is the limit of a sequence of (non-negative) integers and conversely, and also every Cauchy sequence in $\mathbb{Q}$ consisting of integers has a limit in $\mathbb{Z}_p$.

[10]