Assignment

Subject-Mathematics

Semester I

Course-Complex Analysis I

Paper Code-DEMATH1CORE2

Total Marks-25

Group – A

Answer **any one** question from the following questions (15 *marks*)

1. (a) Let f be analytic in $|z| \leq R$. For each $0 < r < R$, let $M(r) = \max_{|z|=r}\{|f(z)|\}$ and $A(r) = \max_{|z|=r}\{Re f(z)\}$. Then prove that

$$M(r) \leq \frac{R+r}{R-r}|f(0)| + \frac{2r}{R-r}A(R).$$

(b) If $f = u + iv$ is an entire function and $u^2 \leq v^2 + 2022$ on $\mathbb{C}$, then prove that f is a constant. [10+5=15]

2. (a) Let f be analytic in a domain D and a be a point in D such that $|f(z)| \leq |f(a)|$ holds for all $z \in D$. Then prove that f is constant.

(b) Prove that for every $f \in \mathcal{H}(\mathbb{C}\backslash\{0\})$ such that $|f(z)| \leq |z|^2 + |z|^{-\frac{1}{2}}$ for all $z \in \mathbb{C}\backslash\{0\}$ is necessarily a polynomial of degree at most two. [10+5=15]

Group – B

Answer **any one** question from the following questions (10 *marks*)

1. Let f be analytic in a simply connected domain D. Then prove that there exists a function F in D such that $F'(z) = f(z)$. In particular, $\int_\gamma f(z)dz = 0$ for each simple closed contour γ in D. [10]

2. Let f be a single valued analytic function on the annulus $\mathcal{A} = \{z \in \mathbb{C} : r_2 < |z-\alpha| < r_1\}$ and let C denote any positively oriented simple closed contour around α and lying in $\mathcal{A}$. Then show that f has a unique representation of the form $f(z) = \sum_{-\infty}^{\infty} a_n(z-\alpha)^n$, $(r_2 < |z-\alpha| < r_1)$, where [10]

$$a_n = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-\alpha)^{n+1}},\ for\ n \in \mathbb{Z}.$$