Assignment

Subject-Mathematics

Semester I

Course-Abstract Algebra

Paper Code-DEMATH1CORE1

Total Marks-25

Group – A

Answer **any one** question from the following questions (15 *marks*)

1. (a) Prove that if G is a finite group of order $p^r m$, where p is a prime, r and m are positive integers, and $gcd(p, m) = 1$, then G has a subgroup of order p^k for all k, $0 \leq k \leq r$.

(b) Let G be a cyclic group of order n and ϕ be the Euler ϕ-function. Prove that $|Aut(G)| = \phi(n)$. [10 + 5 = 15]

2. (a) Prove that every Euclidean domain is a principal ideal domain. Give an example to show that converse is not true in general.

(b) Let M be an ideal of a commutative ring R. Prove that R/M is a field if and only if M is a maximal ideal $x^2 \in M$ implies $x \in M$ for all $x \in R$.

[8 + 7 = 15]

Group – B

Answer **any one** question from the following questions (10 *marks*)

1. (a) Let R be a commutative ring with 1. Prove that the following conditions are equivalent:
(i) R is a field.
(ii) $R[x]$ is a Euclidean domain.
(iii) $R[x]$ is a PID. [10]

2. (a) Let H_1 and H_2 be subgroups of a group G with H_2 normal. Then show that $H_1/H_1 \cap H_2 \simeq (H_1H_2)/H_2$.
(b) Show that $8\mathbb{Z}/56\mathbb{Z} \simeq \mathbb{Z}_7$. [6 + 4 = 10]