Programme Outcomes, Programme Specific Outcomes and Course Outcomes

For

PG Programmes

Program Name: ***PG. in Mathematics***

Number of Semesters: 4

Department of Mathematics
University of North Bengal

West Bengal, INDIA

Program Outcomes

- To develop and conduct continuing education programs for Mathematics graduates with a view to update their fundamental knowledge base and problem-solving capabilities in the various areas of Mathematics.
- Enable students to enhance mathematical skills and understand the fundamental concepts of pure and applied mathematics.
- To inculcate the curiosity for mathematics in students and to prepare them for future research.
- Develop, design and implement research projects competently and independently.
- Identify and define emerging problems related to one's area of interest.

Program Specific Outcomes

- Apply the knowledge of mathematical concepts in interdisciplinary fields.
- Upon completion of the program, students will be able to demonstrate critical understanding at an advanced level with up-to-date knowledge in research methodology of his/her field of interests.
- Students will be completely prepared to take up PhD and continue his/her research.
- Qualify Competitive Exams like NET/GATE/SET/GRE etc.

	Course Code	Course Name	Marks	Credits
SEM: I	C-I1T	Groups and Rings	50	2
	C-I2T	Naïve Set Theory and Elements of Topology	50	2
	C-I3T	Analysis of Several Variables	50	2
	C-I4T	Complex Analysis-I	50	2
	C-I5T	Real Analysis	50	2
	C-I6T	Ordinary Differential Equations	50	2
SEM: II	C-II1T	Linear Algebra	50	2
	C-II2T	Point-Set Topology	50	2
	C-II3T	Differential Geometry	50	2
	C-II4T	Functional Analysis	50	2
	C-II5T	Abstract Measure Space	50	2
	C-II6T	Partial Differential Equations	50	2
SEM: III	E-IIIT1	Measurability and Integration in Abstract Spaces	50	2
	E-IIIT2	Elementary Number Theory	50	2
	E-IIIT3	Complex Analysis –II	50	2
	E-IIIT4	Field Extension and Galois Theory	50	2
	E-IIIT5	Topological Groups	50	2
	E-IIIT6	Algebraic Topology	50	2
	E-IIIT7	Integral Equation and Integral transform	50	2
	E-IIIT8	Differential Topology	50	2
	E-IIIT9	Theory of Approximation	50	2
	E-IIIT10	p-adic Analysis	50	2
SEM: IV	E-IVT1	Signed Measure and Product Measure	50	2
	E-IVT2	Graph Theory	50	2
	E-IVT3	Topological Algebra	50	2
	E-IVT4	General theory of Integration	50	2
	E-IVT5	Modules and Number Theory	50	2
	E-IVT6	Advanced complex Analysis	50	2
	E-IVT7	Discrete Mathematics	50	2
	E-IVT8	Algebraic Geometry	50	2
	E-IVT9	Category Theory	50	2
	S-IV6P	Numerical problem solving by computer Programming (PRACTICAL)	50	2

SEMESTER—I		
Course Code	**Course Name**	**Course Outcomes**
C-I1T	Groups and Rings	Knowledge gained: • Concept of group action. • Class equation and its applications. • Sylow's Theorems and their applications. • Ideal of a ring, Polynomial rings. • EDs, PIDs, & UFDs and relations among them. Skills gained: • Solving problems using the powerful concept of group action. • Finding the number of subgroups, normal subgroups of a finite group. • Ability to understand a large class of commutative rings by regarding them as quotients of polynomial rings by suitable ideals. Competency developed: • Applying the concept of a group action to real life problems such as Counting. • Facility in handling problems involving polynomial equations. • Facility in working with situations involving commutative rings. • Ability to understand the various PID's whose common example is the ring of integers $\mathbb{Z}$.
C-I2T	Naïve Set Theory and Elements of Topology	Knowledge gained: • Set as a domain of thought in Mathematical Sciences • Conversion of Necked Set into Mathematical structure • Cardinality of Sets (that can distinguish even infinite sets having huge difference in some sense) and Ordinal Numbers • A fast journey from Metric space to Topological Space • How roots of analysis are spread within Topological spaces • Equality in Topological sense and its relevance • Properties of Topological Spaces (Separation Axioms, Compactness, Connectedness) • Inadequacy of sequence and its recovery by Nets and Filters. Skills gained: • Analysis on Abstract Spaces to some extent • Analysis without concepts of length or distance Competency gained: • Ability of real/complete understanding of analysis and feelings of identity in differences
C-I3T	Analysis of Several Variables	Knowledge gained: • Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear operator. • Jacobian matrix. • Chain rule. Notions of partial derivatives. • Inverse and implicit function theorems. • Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. • Fubini's theorem. • Partitions of unity. • Change of variables and the Jacobian formula. • Picard's Theorem, Green's theorem, Gauss (Divergence) theorem and Stoke's theorem.

		Skills gained: • Generalization of concept of differentiability. • Generalization of concept of integrability. • Generalization of theorems. Competency gained: • Ability to solve higher dimensional derivatives. • Ability to solve higher dimensional integration. • Apply inverse and implicit function theorems.
C-I4T	Complex Analysis-I	Knowledge gained: • Complex plane as a metric space, Analytic functions, Power series as analytic function, Cauchy-Riemann differential equations. • Cauchy's theorem, Cauchy's integral formula and applications, Morera's theorem, Liouville's theorem, Zeros of analytic function and related properties. • Maximum modulus theorem and its applications, Singularities. Skills gained: • Differentiation of functions on C, deciding if a function on C is analytic. • Development of analytic functions into power series. • Range of a bounded entire function. Competency developed: • Understanding of topological and geometric properties of the complex plane. • Differentiation and integration of functions on C. • Location of maximum and minimum points of an analytic function.
C-I5T	Real Analysis	Knowledge gained: • Concept of extended real numbers, Lebesgue and Borel measures on real line. • Measurability of real sets. • Measurability of extended real valued functions. • Foundation of extension to abstract spaces. Skill gained: • Solving problems relating to determinations of measures of finite, infinite sets. • Ability of constructing different Borel sets. • Ability of constructing measurable, non-measurable sets and functions. Competency developed: • Applying results to the later topics, namely in abstract spaces. • Applying the notions for the study of subtle concepts like Cantor sets.
C-I6T	Ordinary Differential Equations	Knowledge gained: • Solution methods for first and second order equations. • Power series solutions. • Properties of Bessel functions and Legendre polynomials. • Existence and uniqueness of initial value problems. • Picard's and Peano's theorems, Gronwall's inequality. • Maximal interval of existence, continuous dependence. • Higher order linear equations and linear systems, fundamental solutions, Wronskian, matrix exponential equations. • Boundary value problems for second order equations, Green functions, eigen value problems. Skill gained: • Solution method for higher order equations. • Power series solutions. • Reducing linear system of equations into matrix differential equation form. • Computing Wronskian and fundamental solutions. • Constructing Green functions.

		• Solving eigenvalue problems. Competency developed: • Solving higher order equations, qualitative analysis of special functions. • Understanding of linear system of equations, Green functions, eigenvalue problems.
SEMESTER—II		
Course Code	**Course Name**	**Course Outcomes**
C-II1T	Linear Algebra	Knowledge gained: • Matrix theory, determinants and their application to systems of linear Equations • Eigenvalues, diagonalization of matrices and reduction of systems of linear equations into simpler systems of easily tractable nature. • Vector theory: subspace, basis, linear independence, inner product spaces etc. • Applications of matrix algebra. Skills gained: • Matrix manipulations. • Handing of systems of linear equations. • Use mathematical software to solve problems on linear systems. • Ability to go abstract from concrete: from concrete notion of solution spaces to vector spaces. • Linear modelling problems. Competency developed: • Solving Systems of linear equations. • Qualitative analysis of systems of linear equations. • Vector Spaces, linear independence and foundations of abstract algebraic thinking.
C-II2T	Point-Set Topology	Knowledge gained: • Observation of differences between metric space and Topological Space via different kinds of compactness of Topological spaces. • Role of ordinals to distinguish different compactness of Topological spaces. • Paracompactness of Topological Spaces towards searching stronger to simpler condition for metrizability of Topological Spaces. • Local compactness and compactification to embed possible Topological spaces into well behaved compact Hausdorff spaces. • Covering spaces and Uniform spaces (which placed in between metric space and topological space according to their properties/ behavior). Skills gained: • Analysis in abstract spaces. • Simplifications of proofs or understanding by using compactification. Competency gained: • Ability to learn/understand any topic/subject related to topology.
C-II3T	Differential Geometry	Knowledge gained: • Curves, surfaces, surface patches. • Parameterizations of curves and surfaces, • Different surfaces like smooth surfaces, regular surface, orientable surfaces etc. • Differential maps between regular surfaces. • The first and second fundamental forms. • Gauss map • Normal curvatures, principle curvatures, Gaussian and mean curvatures. Skills gained: • Study different curves and surfaces and its natures.

		• Verify smooth surfaces, regular surface, orientable surfaces etc. • Construct differential maps between smooth surfaces • Evaluate 1st and 2nd fundamental forms of surface patches. Competency developed: • Realizing the behaviors of different surfaces. • Analyze different curves and surfaces. • Characterize different surfaces.
C-II4T	Functional Analysis	Knowledge gained: • Definition of normed linear spaces, Banach spaces and examples. • Quotient space of normed linear spaces. • Equivalent norms. • Riesz's Lemma. • Basic properties of finite dimensional normed linear spaces. • Concept of bounded linear operators between these spaces. • Concept of the spectrum of a bounded linear operator. • Concept of the dual space of a normed linear space. • Uniform boundedness theorem and some of its consequences. • Open mapping and closed graph theorems. • Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces. • Reflexive spaces. • Definition of inner product spaces, Hilbert Spaces and examples. • Orthonormal sets. Bessel's inequality. • Complete orthonormal sets and Parseval's identity. • Riesz representation theorem. • Adjoint of an operator on a Hilbert space. • Reflexivity of Hilbert spaces. • Concept of compact, Positive, projection, self-adjoint, normal and unitary operators. Skills gained: • Equivalent norms on a vector spaces define the same topology. • On a finite dimensional vector space any two norms are equivalent. • Using topology to work with infinite dimensional vector spaces. • Viewing C[a,b] with sup norm and integration norm respectively as Banach space and incomplete norm linear space. • Comparing the differences between finite and infinite dimensional spaces. • Comparing the differences between Banach and Hilbert spaces. • Analyzing the structure of the spectrum of certain operators. Competency developed: • Comparing the differences between basis and Schauder basis. • Working with a complete orthogonal set in a Hilbert space. • Fredholm and other integral operator as a linear operator. • Investigating the best approximation of a given vector by vectors in a given subspace. • Computing the dual spaces of certain Banach spaces. • Working with weak and weak * topologies on normed linear spaces.
C-II5T	Abstract Measure Space	Knowledge gained: • Concepts of measure spaces and measurable spaces. • Borel σ-algebra. • Cantor ternary set. • Completion of measure spaces. Skills gained: • Ability of defining measurability from arbitrary outer measure. • Ability of understanding central importance of Cantor Lebesgue functions. Competency developed: • Realizing concept of measure for abstract sets.

		• Ability to obtain completion of measure spaces.
C-II6T	Partial Differential Equations	Knowledge gained: • Cauchy problem for 1st order hyperbolic equations, method of characteristics. • Classification of 2nd order partial differential equations, normal forms and characteristics. • Lagrange-Green's identity, uniqueness by energy methods. • Mean value property, weak and strong maximum principle, Green's function, poisson's formula, Dirichlet's principle. • Initial value problem for heat equation, fundamental solution, weak and strong maximum principle and uniqueness results. • Uniqueness of wave equations, D'alembert's principle, Duhamel's principle. Skill gained: • Solving Cauchy problem. • Solving 1st and 2nd order partial differential equations. • Uniqueness of heat and wave equations. • Methods of separation of variables for Laplace, heat and wave equations. Competency gained: • Reduction into canonical form and solving partial differential equations. • Solution of heat, wave and Laplace equations

SEMESTER—III

Course Code	Course Name	Course Outcomes
E-IIIT1	Measurability and Integration in Abstract Spaces	Knowledge gained: • Concepts of simple functions • Integration of functions on arbitrary measure space • Regular only measure and metric outer measure Skills gained: • Generalization of classical Lebesgue integral on real sets • Integration of bounded functions on sets of finite measure Competency developed: • Ability of applying the concepts of integration for the study in subsequent chapters namely, signed and product measure.
E-IIIT2	Elementary Number Theory	Knowledge gained: • Division algorithm and Euclidean algorithm. • Congruence Chinese Remainder Theorem. • Diophantine equations. • Quadratic reciprocity Law. • Distribution of primes. • Arithmetical functions. Skills gained: • Use of congruence as a tool to reduce a hard labour of work in some calculations. • Solving a Diophantine equation and system of Diophantine equations. • Finding primitive roots. • Establishing existing identities using Mobius inversion formula. Competency developed: • Useful tools in various areas of number theory, viz. analytic and algebraic number theory, cryptography, modular forms etc.
E-IIIT3	Complex Analysis –II	Knowledge gained: • Cauchy's residue theorem, and applications.

		• Argument Principle, Rouche's theorem. • Bilinear transformation, Principle of symmetry, Conformal mapping. • Analytic continuation and different methods of analytic continuation. Skills gained: • Classifying singularities. Integration of functions on C, applications to counting zeros and poles. • Evaluation of definite real integrals. • Constructing Mobius transformation between regions. Competency developed: • Viewing analytic function as conformal mapping. • Extending the domain of an analytic function. • Applications to problems from real analysis.
E-IIIT4	Field Extension and Galois Theory	Knowledge gained: • Concept of algebraic element, transcendental elements, minimal polynomials etc. • Various field extensions, e.g. splitting fields, algebraically closed fields, perfect fields, normal fields, separable fields, cyclotomic fields etc. • Concept of Galois groups and Galois extensions. Skills gained: • Construction of minimal polynomials, splitting fields, finite fields etc. • Verify/identify normal fields, separable fields. • Construction of algebraic numbers geometrically using straightedge and compass only. • Formulation of Galois groups and Galois field extensions. Competency developed: • Relate Structures of fields with certain related groups. • Abel's famous theorem on the insolvability of the general quintic polynomials.
E-IIIT5	Topological Groups	Knowledge gained: • How to mix algebraic group structure with topological structure and its application Skills gained: • Use of topological properties to know algebraic properties • Topological properties via algebraic properties. Competency gained: • Applications of Point-Set Topology towards extension of the study of Algebra.
E-IIIT7	Integral Equation and Integral transform	Knowledge gained: • Construction of Green's function. • Concept of various Integral Equations: Fredholm and Voltera type. • Calculus of Variations, Euler-Lagrange's equations. • Convolutions and applications. • Distributions and their transforms. • Various integral transforms: Applications to Wave, Heat and Laplace Equations. Skills gained: • Obtain solution of a boundary value problem using integral equations. • Obtain minimum surface of revolution from a variational formulation. • Solution of Wave, Heat and Laplace equations using integral transform technique. Competency developed: • Solutions of the Abel's Integral Equations.

		• Handle separable and symmetric kernels of an integral equation. • Solution of Isoperimetric and Brachistochrone problems using variation method. • Applying convolution theorem on the transformed function in order to get the primitive.
E-IIIT9	Theory of Approximation	Knowledge gained: • Definition Concept of best approximation in a normed linear space. • Convexity-uniform convexity, strict convexity and their relations. • Weierstrass theorem, Korovin theorem. • Bernstein polynomials. • Bernstein's inequality. • Jacson's theorems. • Haar uniqueness theorem. • Simultaneous approximation. • L^p-approximation. Skills gained: • Hilbert space is strictly convex and the space C[a,b] with supnorm is not a strictly convex. • Extremal point. • Chebyshev polynomials. • Lipschitz class. • Approximation by means of Fourier Series. Competency developed: • Apply uniform convexity and strict convexity notions. • Apply Weierstrass and Korovin theorems. • Apply Jacson's theorem and Haar uniqueness theorem.

SEMESTER—IV

Course Code	Course Name	Course Outcomes
E-IVT1	Signed Measure and Product Measure	Knowledge gained: • Concepts of signed measure on a σ-algebra. • Decomposition of signed measure space and signed measure. • Absolute continuity and mutual singularity. • Product measure spaces. Skill gained: • Hahn Decomposition, Jordan decomposition, and Lebesgue decomposition. • Solving problems on absolute continuity and mutual singularity. • Calculate product measure by integrals. Competency gained: • Realizing importance of Radon-Nykodim theorem. • Fubini's theorem and Tonell'is theorem and their application.
E-IVT2	Graph Theory	Knowledge gained: • Definition of Graph, subgraph their representations, degree and algebraic operations. • Connected graphs, weighted graphs and shortest paths • Trees: Characterizations, spanning tree, minimum spanning trees • Eulerian and Hamiltonian graphs: Characterization, Necessary and sufficient conditions • Special classes of graphs: Bipartite graphs, line graphs, chordal graphs. • Vertex and edge coloring • Directed graphs: Out-degree, in-degree, connectivity, orientation

		Skills gained: • Develop algorithm for shortest path between two vertices • Construct matching problem using bipartite graph and hence define a problem related to perfect matching. • Applying Fleury's algorithm to construct Eulerian graph. • Applying Kruskal's algorithm to construct minimum spanning tree. • Construct suitable techniques to color a graph Competency gained: • Classify Class-1 graphs and class-2 graphs in the light of edge coloring. • Construct perfect matching between two given sets of boys and girls with different cardinality. • Using the concept of connectivity of directed graph, construct electrical circuits. • In Google Maps, various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find the shortest path between two nodes.
E-IVT4	General theory of Integration	Knowledge gained: • Concepts of Tagged Gauge Partition. • Intrinsic power of tagged partition. • Henstock-Kurzweil Integral. • Important properties including Saks-Henstock Lemma and Vitali covering Theorem. Skills gained: • Understanding the use of Tagged partition and its applications in continuity. • Using Right-Left procedure appropriately. • Understanding basic properties. Competency developed: • Viewing Heustock integral as the highest possible generalized integral on $\mathbb{R}$. • Realising that HK-integration corrects the defects of Classical Riemann Theory and both simplifies and extends Lebesgue Theory.
E-IVT5	Modules and Number Theory	Knowledge gained: • Modules as vector spaces over rings. • Special classes of modules: free modules, simple modules, cyclic modules etc. • Theory of modules over PID and its application to Jordan and Rational canonical forms. • Number fields, quadratic and cyclotomic fields. • Characters of a finite abelian group, Dirichlet Characters. • Equivalent forms of Prime Number Theorem. Skills gained: • Ability to deal with modules which is indispensable in wide ranges of mathematical disciplines such as algebra, topology, number theory, operator theory etc. • Ability to handle modern algebraic notions like quotients, generators and relations etc. • Ability to deal with different problems in Number theory such as problems related to primes in an Arithmetic Progression, L-functions etc. Competency developed: • Basic preparation for research in various areas of pure mathematics like algebraic geometry, Algebraic Number Theory, Analytic Number Theory etc. • Understanding of Prime Number Theorem.

E-IVT6	Advanced Complex Analysis	Knowledge gained: • Mean value property, Poisson's integral formula, Dirichlet's problem for a disc. • Infinite product and its convergence. • Factorization theorems, Picard theorems. • Poisson-Jensen formula, Nevanlinna's 1st fundamental theorem. Skills gained: • Relation between harmonic function and analytic function. • Generalization of fundamental theorem of algebra. • Range of an entire function. Competency developed: • Characterizing harmonic functions. • Representing entire functions as infinite product. • Constructing maximum modulus like function for meromorphic functions.
E-IVT7	Discrete Mathematics	Knowledge gained: • Basic combinatorics, induction, inclusion exclusion, pigeon-hole principle. • More advance topics in combinatorics: recurrence relations, generating functions. • Application to real life problems such as network theory, data structure, optimization etc. • Basic logical concepts, analyzing arguments, quantification theory. • Construction of method of deduction for validity of truth. Skills gained: • Efficiency in handling with discrete structures. • Efficiency in solving concrete combinatorial problems whose presence is ubiquitous in science and engineering. • Proving validity and invalidity of arguments. • Efficiency in handling with universal and existential quantifiers. Competency developed: • Ability to use graphs as unifying theme for various combinatorial problems. • Ability to apply combinatorial intuitions in network theory, data structure and various other fields of science. • Understanding of symbolic logic.
E-IVT9	Category Theory	Knowledge gained: • Concept of category, Morphisms. • Functor, forgetful functor, faithful functor. • Product category. • Natural transformation. • Representable functor, embedding. • Yoneda's lemma and its applications. • Adjoint functor. Initial object, terminal object. • Limit, colimit, pull back diagram, and push out diagram. Skills gained: • Construction of different functors. • Ability to study different categories. • Calculating natural transformations, limit, colimits etc. Competency developed: • Study the high-level abstractions such as sets, rings, and groups. • Philosophical investigations of concepts such as space, system and even truth. • Study of logical systems.
S-IV6P	Numerical problem solving by computer	Knowledge gained: • Basic ideas about computer programming language. • Fundamental data types, operators and expressions, conditional branching used in C. • C-functions (declaring and calling a function), arrays (one dimensional

	Programming (PRACTICAL)	and multidimensional), pointers (Accessing array elements through pointers). • Opening and closing a file, reading from a file and writing to a file. • Solving numerical problems using C-programming. Skills gained: • Efficiency in handling with data types, C-operators, expression in C, conditional branching, looping. • Construct C-functions, use of Standard C library functions • Efficiency in handling with arrays, pointers, C-file. • Efficiency in solving numerical problems such as interpolation, differentiation, integration, matrix problem, ODE, PDE etc. using C-programming. Competency developed: • Ability to understand syntax in C (data types, arrays, pointers, C-files, C-functions, etc.) • Ability to solve various numerical problems occurring in applied mathematics, theoretical physics, and biological science.