UNIVERSITY OF NORTH BENGAL

समानो मन्त्रः समितिः समानी

Syllabus and Examination Scheme for

M. Sc.

in

MATHEMATICS

Under

CHOICE BASED CREDIT SYSTEM (CBCS)

(With effect from Academic Session 2021-2022)

Department of Mathematics, University of North Bengal, Raja Rammohunpur, P.O.- N.B.U., Dist- Darjeeling, West Bengal, India, Pin-734013

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06/09/2021.

Signature of HOD
Department of Mathematics, NBU

HEAD
Department of Mathematics
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Structure of Syllabus for M.Sc. in Mathematics

Semester-I

Course Code	Course Name	Marks (Term End)	Marks (Internal)	Duration of Exam (Hrs.)	Total Lectures (Hrs.)	Credit
MATH-CCT-101	**Groups and Rings**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-102	**Naive Set Theory and Elements of Topology**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-103	**Analysis of Several Variables**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-104	**Real Analysis**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-105	**Complex Analysis**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-106	**Ordinary Differential Equations and Special Functions**	**50**	**10**	**2**	**50**	**2.4**
MATH-CET-107	**Project-I**	**-**	**40**	**-**		**1.6**
	Total	**400**		**-**		**16**

Semester-II

Course Code	Course Name	Marks (Term End)	Marks (Internal)	Duration of Exam (Hrs.)	Total Lectures (Hrs.)	Credit
MATH-CCT-201	Modules and Linear Algebra	50	10	2	50	2.4
MATH-CCT-202	Point-Set Topology	50	10	2	50	2.4
MATH-CCT-203	Functional Analysis	50	10	2	50	2.4
MATH-CCT-204	Classical Mechanics and Calculus of Variation	50	10	2	50	2.4
MATH-CCT-205	Integral Equations and Integral Transforms	50	10	2	50	2.4
MATH-CCT-206	Partial Differential Equations	50	10	2	50	2.4
MATH-CET-207	Project-II	-	40	-		1.6
	Total	400		-		16

Semester-III

Course Code	Course Name	Marks (Term End)	Marks (Internal)	Duration of Exam (Hrs.)	Total Lectures (Hrs.)	Credit
MATH-CCT-301	**Differential Geometry and Its applications**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCT-302	**Field Extension and Galois Theory**	**50**	**10**	**2**	**50**	**2.4**
MATH-ECT-303	**Elective:** Any two papers to be chosen from **Appendix-I**	**50 + 50**	**10 + 10**	**2 + 2**	**50 + 50**	**2.4 + 2.4**
MATH-ECT-304	**Elective:** Any two papers to be chosen from **Appendix-II**	**50 + 50**	**10 + 10**	**2 + 2**	**50 + 50**	**2.4 + 2.4**
MATH-CET-305	**Project-III**	**-**	**40**	**-**		**1.6**
	Total	**400**		**-**		**16**

Semester-IV

Course Code	Course Name	Marks (Term End)	Marks (Internal)	Duration of Exam (Hrs.)	Total Lectures (Hrs.)	Credit
MATH-CCT-401	**Graph Theory, Algorithms, and Combinatorics**	**50**	**10**	**2**	**50**	**2.4**
MATH-CCP-402	**Numerical Problem Solving by Computer Programming (PRACTICAL)**	**-**	**60**	**4(1+3)**	**50**	**2.4**
MATH-ECT-403	**Elective:** Any two papers to be chosen from **Appendix-III**	**50 + 50**	**10 + 10**	**2 + 2**	**50 + 50**	**2.4 + 2.4**
MATH-ECT-404	**Elective:** Any two papers to be chosen from **Appendix-IV**	**50 + 50**	**10 + 10**	**2 + 2**	**50 + 50**	**2.4 + 2.4**
MATH-CET-405	**Term Paper**	**-**	**40**	**-**		**1.6**
	Total	**400**		**-**		**16**

In each semester, every student will undertake the following tasks: report writing, presentation and open viva-voce before the faculty members

Tasks	Contents	Marks
Project-I	Report: Any topic to be chosen from course and curriculum	10
	Presentation on the report	10
	An open viva-voce	20
Project-II	Report: Continuation of project-I	10
	Presentation on the report	10
	An open viva-voce	20
Project-III	Report: A specific topic, probably at advance PG level, will be offered	10
	Presentation on the report	10
	An open viva-voce	20
Term Paper	Report: A review work based on a research topic will be offered	10
	Presentation on the report	10
	A grand viva-voce will be conducted in presence of one/two external examiner(s).	20

Appendix-I

Elective Papers for MATH-ECT-303 (M1 to M8)	
Elective Paper Sub-Code	Title of the Paper
M1	Measurability and Integration in Abstract Spaces
M2	Topological Groups
M3	Algebraic Topology
M4	Elementary Number Theory
M5	Advanced Complex Analysis-I
M6	Advanced Functional Analysis-I
M7	Theory of Approximation
M8	p-adic Analysis

Appendix-II

Elective Papers for MATH-ECT-304 (N1 to N8)	
Elective Paper Sub-Code	**Title of the Paper**
N1	**Advanced Numerical Analysis**
N2	**Continuum Mechanics**
N3	**Computational Partial Differential Equations**
N4	**Dynamical System**
N5	**Fluid Mechanics**
N6	**Numerical Programming in Computational Software**
N7	**Statistical Learning**
N8	**Quantum Mechanics**

Appendix-III

Elective Papers for MATH-ECT-403 (M1 to M11)	
Elective Paper Sub-Code	**Title of the Paper**
M1	**Signed Measure and Product Measure**
M2	**Topological Algebra**
M3	**Differential Topology**
M4	**Analytic Number Theory**
M5	**Advanced Complex Analysis-II**
M6	**Advanced Functional Analysis-II**
M7	**Advanced Algebra**
M8	**Modular Forms**
M9	**Algebraic Geometry**
M10	**Category Theory**
M11	**General Theory of Integration**

Appendix-IV

Elective Papers for MATH-ECT-404 (N1 to N7)	
Elective Paper Sub-Code	**Title of the Paper**
N1	**Boundary Integral Equations**
N2	**Mathematical Ecology**
N3	**Biofluid Mechanics**
N4	**General Theory of Relativity and Cosmology**
N5	**Lie Theory of Ordinary and Partial Differential Equations**
N6	**Nonlinear Optimization**
N7	**Computational Statistics**

Question Pattern

Question Pattern for all except Paper MATH-CCP-402	Group-A (10 Marks)	5 Moderate problems each of 2 marks
	Group-B (10 Marks)	2 Harder problems out of 3 each of 5 marks
	Group-C (30 Marks)	3 Questions out of 5 each of 10 marks
Question Pattern for the Paper MATH-CCP-402	Written Test (25 Marks)	• 3 questions out of 5 each of 5 marks • 1 question out of 2 each of 10 marks
	LAB Test (35 Marks)	• Practical Note Book: 5 • Viva-voce: 5 • Solving 1 problem of 5 marks and 2 problems of 10 marks by computer programming: 25

<u>Detailed Syllabus</u>

MATH-CCT-101: Groups and Rings **Marks: 50+10=60 Credit: 2.4**
Total Lectures: 50 Hrs.

Groups: Direct product of groups, Finite Abelian groups, Group action, Class equation, Cauchy's theorem, Sylow's theorems, Generalized Cayley's theorem.
Simple groups, Solvable groups, Nilpotent groups, Simplicity of alternating groups, Normal and subnormal series, Composition series, Jordan–Holder theorem, Semi direct product, Free groups, Free Abelian groups.
Rings: Maximal ideals, Prime ideals, ED, PID, UFD.
Polynomial rings, Division algorithm in polynomial rings, Irreducibility of Polynomials, Eisenstein's criterion of irreducibility, Noetherian Rings. Artinian Rings, Hilbert Basis Theorem, Primary ideals, Primary decomposition theorem.

<u>Recommended Books:</u>

1. *D. S. Dummit, R. M. Foote, Abstract Algebra (3e), John Wiley and Sons.*
2. *S. Lang, Algebra, 3rd Edition, Springer (Indian reprint 2004).*
3. *M. K. Sen, S. Ghosh, P. Mukhopadhyay, S. K. Maity, Topics in Abstract Algebra, Universities Press.*
4. *J. R. Gallian, Contemporary Abstract Algebra, Narosa Publishing House.*
5. *J. B. Fraleigh, A First Course in Abstract Algebra, Narosa Publishing House.*
6. *M. Artin, Algebra, Prentice Hall.*
7. *I.N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.*
8. *D. S. Malik, J. N. Mordeson, M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill, International Edition, 1997.*

MATH-CCT-102: Naive Set Theory and Elements of Topology
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

Axiom of choice and existence of choice function. Partially ordered set, linearly ordered set, well ordered set and product of the same kinds, Zorn's lemma, well ordering principle with special emphasis on Ordinal and Cardinal numbers.
Topological spaces, open and closed sets, basis and sub-basis, closure, interior and boundary of a set. Subspace topology. Continuous maps: properties and constructions; Pasting Lemma. Open and closed maps, Homeomorphisms. Product topology, Quotient topology and examples of Topological Manifolds. Countability and separation axioms: Urysohn's lemma, Tietze extension theorem and applications. Urysohn embedding lemma and metrization theorem for second countable spaces. Connected, path-connected and locally connected spaces. Lindelof and Compact spaces. Net, Filters.

<u>Recommended Books:</u>

1. *J. R. Munkres, Topology: a first course, Prentice-Hall (1975).*
2. *G. F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw-Hill (1963).*
3. *M. A. Armstrong, Basic Topology, Springer.*
4. *J. L. Kelley, General Topology, Springer-Verlag (1975).*
5. *J. Dugundji, Topology, UBS (1999).*
6. *S. Willard, General Topology, Dover (2004).*
7. *I. P. Natanson, Theory of functions of a real variable, Vol. II. (especially for Ordinal numbers)*

MATH-CCT-103: Analysis of Several Variables Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Topology of $\mathbb{R}^n$, $GL_n(\mathbb{R})$etc.. Differentiability of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$ and the derivative as a linear map. Determinant as mapping; its continuity and differentiability. Existence and meaningfulness of e^A and its continuity as well as differentiability (A is a real square matrix).
Higher derivatives, Chain Rule, mean value theorem for differentiable functions, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier, Sard's theorem. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals.
Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustration with plenty of examples. Inverse and implicit function theorems. Picard's Theorem.
Curves in $\mathbb{R}^2$ and $\mathbb{R}^3$. Line integrals, Surfaces in $\mathbb{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green's theorem, Gauss Divergence theorem and Stoke's theorem.

<u>Recommended Books:</u>

1. *M. Spivak, Calculus on Manifolds, Benjamin (1965).*
2. *W. Rudin, Principles of mathematical Analysis, Mc Graw-Hill.*
3. *T. M. Apostol, Mathematical Analysis, Addison-Wesley,(2007)*
4. *J. Munkres, Analysis on Manifolds, CRC Press, (2018)*
5. *T. M. Apostol, Calculus (Vol 2), John Wiley.*

MATH-CCT-104: Real Analysis Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Extended real numbers, algebraic operations and convergence in extended real number system, Lebesgue outer measure on R, elementary properties of Lebegue measure space including σ-finiteness, translation invariance, positive homogeneity, existence of non-Lebesgue measurable sets, regularity of Lebesgue outer measure, Borel measurability on R, measurable functions, operations with measurable functions, sequence of measurable functions, Cantor ternary set and Cantor-Lebesgue function.
Abstract measure spaces, σ-algebra of sets, limits of sequences of sets, Borel σ-algebra, measure on σ-algebra, measurable spaces and measure spaces, monotone convergence theorems for sequences of measurable sets.

<u>Recommended Books:</u>

1. *S K. Berberian, Fundamentals of Real Analysis, Springer.*
2. *G. De Barra, Measure Theory and Integration, New Age International Publ.*
3. *H. L. Royden, Real Analysis, Prentice-Hall Of India Pvt. Limited, (1988).*
4. *W. Rudin, Principles of Mathematical Analysis, McGraw-Hill, (2013).*
5. *J. Yeh, Lectures on Real Analysis, World Sci.*
6. *R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, (1966).*

MATH-CCT-105: Complex Analysis **Marks: 50+10=60 Credit: 2.4**

Total Lectures: 50 Hrs.

Complex integration, Winding number or Index of a closed curve, Homotopy version of Cauchy's Theorem, Morera's theorem, primitives of analytic functions, Zeros of an analytic function, Open mapping theorem, Inverse function theorem, Singularities and their classification, Riemann's theorem, Limit points of zeros and poles, Casorati-Weierstrass's theorem, behaviour of a function at the point at infinity, Theory of residues, Cauchy's residue theorem, evaluation of improper integrals, Argument principle, Rouche's theorem and its applications, Maximum modulus theorems, Schwarz lemma, Conformal mappings, Möbius transformation, Principle of symmetry, introduction to Analytic continuation, Monodromy theorem.

Recommended Books:

1. *J. B. Conway, Functions of one complex variable, 2nd Ed., Narosa Publishing House, New Delhi, 1997.*
2. *R. Remmmert, Theory of complex functions, Springer-Verlag, New York, 1991.*
3. *L. V. Ahlfors, Complex Analysis- 3rd Edn, McGraw-Hill, 1979*
4. *R. V. Churchill and J. W. Brown, Complex Variables and applications, McGraw Hill, 1996.*
5. *I. Markushevich, Theory of Functions of a Complex Variable (Vol. I, II & III), Prentice-Hall, 1965 & 1967.*
6. *E. C. Titchmarsh, The Theory of Functions, Oxford University Press, 1939.*
7. *E. T. Copson, Introduction to the Theory of Function of a Complex Variable, Oxford University press, 1970.*
8. *W. Rudin, Real and Complex Analysis, Tata Mc Graw-Hill Education, 1987.*

MATH-CCT-106: Ordinary Differential Equations & Special functions

Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

Review of solution methods for first order as well as second order equations, Existence and Uniqueness of solutions of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, wellposedness.

Higher Order Linear Differential Equations: Fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Boundary Value Problems for Second Order Equations: Ordinary differential equations of the Strum Liouville type and their properties, application to Boundary Value Problems, eigenvalues and eigenfunctions, orthogonality theorem, expansion theorem, Green's function and its properties, Green's function for ordinary differential equations, application to Boundary Value Problems, adjoint equation of n-the order, Lagrange's identity, solution of equation from the solution of its adjoint equation, self-adjoint equation.

Series Solution: Homogeneous linear differential equations, fundamental system of integrals, singularity of a linear differential equation, solution in the neighbourhood of a singularity, regular singularity, equation of Fuchsian type, series solution by the method of Frobenius.

Hypergeometric Equation: Solution near zero, one and infinity, integral formula, hypergeometric functions, properties of hypergeometric function.

Bessel Equation: Solution of Bessel's equation, Bessel function and its properties, generating function, integral representation of Bessel's function, Hankel functions, recurrence relations, asymptotic expansion of Bessel functions.

Legendre Equation: Solution of Legendre equation, Legendre functions, Generating function, Legendre functions of the first and second kinds, Laplace integral, Legendre polynomials, orthogonality, recurrence relation, Schlaefli's integral.
Qualitative Description of Differential Equations: Systems of first order equations and the matrix form, autonomous system, critical points, orbits, trajectories, basic concepts and definitions of phase plane, linearization around critical point, stability, asymptotic stability.

Recommended Books:

1. *S. L. Ross, Differential Equations, 3rd Edn., Wiley India, (1984).*
2. *G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata-McGrawHill, (2003).*
3. *M. Brown, Differential Equations and Their Applications, Springer, (1983).*
4. *W. Boyce, R. Diprima, Elementary Differential Equations and Boundary Value Problems, Wiley, (2009).*
5. *E. A. Codington, Theory of Ordinary Differential Equations, Dover Publications, (2012).*
6. *I. N. Sneddon, Special Functions of Mathematical Physics & Chemistry, Oliver & Boyd, London.*
7. *N. N. Lebedev, Special Functions and their Applications, Dover Publications, (1965).*
8. *E. D. Rainville, Special Functions, Chelsea Publishing Company, (1971).*
9. *S. H. Strogatz, Nonlinear Dynamics & Chaos, CRC Press, (2019).*

MATH-CCT-201: Modules and Linear Algebra Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Modules: Modules over a commutative ring with identity, Submodules, Operations on submodules, Direct sum and Direct product of submodules, Module homomorphisms, Quotient modules, Finitely generated modules, Free modules, Torsion-free modules, Modules over PID, Fundamental structure theorem for finitely generated modules over a PID.
Linear Algebra: Annihilating polynomials, minimal polynomial, Direct sum decompositions, Invariant subspaces, Primary decomposition theorem, Diagonalizations, Triangularizations, Jordan blocks, Jordan forms, Rational canonical form, Generalized Jordan form over an arbitrary fields, Quadratic forms, Reduction and classification of quadratic forms, Bilinear forms, Symmetric bilinear forms, Skew symmetric bilinear forms.

Recommended Books:

1. *K. Hauffman, R. Kunz, Linear Algebra, Pearson Education (INDIA), 2003.*
2. *G. Strang, Linear Algebra and Its Applications, 4th Edition, Brooks/Cole, 2006.*
3. *S. Lang, Linear Algebra, Springer, 1989.*
4. *D. S. Dummit and R. M. Foote, Abstract Algebra (3e), John Wiley and Sons.*
5. *T. Hungerford, Algebra, Springer GTM.*
6. *I. N. Herstein, Topics in Abstract Algebra, Wiley Eastern Limited.*
7. *D. S. Malik, J.M. Mordeson, M.K. Sen, Fundamentals of Abstract Algebra, The McGraw-Hill.*

MATH-CCT-202: Point-Set Topology Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Compact spaces, sequentially compact spaces, limit point compact spaces, countably compact spaces and their basic properties (continuous image, productivity, hereditary property etc.),

relation among themselves. Complete metric spaces and totally boundedness. The Lebesgue number lemma. Coincidence of all above mentioned compactness in metric spaces.
Local compactness. Product of locally compact spaces. Compactiification with special care of Stone-Čech Compactification.
Locally finiteness. σ-locally finiteness. Refinement. Nagata-Smirnov Metrization Theorem. Paracompactness. Micheal's theorem, Paracompactness of metrizable spaces (Stone's theorem), Partition of unity. Locally metrizable spaces. Smirnov metrization theorem.
Function spaces and Uniform Spaces: The compact-open topology. Continuity of composition; the evaluation map. Cartesian Products. Application to Identification Topologies. Basis for Z^Y. Compact subsets of Z^Y. Sequential convergence in the c-Topology. Metric Topologies; Relation to the c-Topology. Pointwise convergence. Comparison of Topologies in Z^Y.
Entourages. Expression of uniform continuity, uniform convergence and Cauchy sequences in terms of entourages. Basic properties of the family of all entourages. Uniformities. Uniform spaces. Base and subbase for uniformities. Union and intersection of uniformities. Uniform topology. Uniform continuity. Uniform isomorphism. Uniform covers. Uniform products and subspaces. Uniformizable spaces. Metrizability of uniformity. Complete uniform spaces and completion. Uniformity generated by proximity.

<u>Recommended Books:</u>

1. *J. R. Munkres, Topology: A first course, Prentice-Hall (1975).*
2. *G. F. Simmons, Introduction to Topology and Modern Analysis, TataMcGraw-Hill (1963).*
3. *S. Willard, General Topology, Dover (2004).*
4. *J. L. Kelley, General Topology, Springer-Verlag (1975).*
5. *J. Dugundji, Topology, UBS (1999).*
6. *K. D. Joshi, Introduction to general topology,* John Wiley & Sons, Inc., New York (*1983). I. P. Natanson, Theory of functions of a real variable, Vol. II. (especially for Ordinal numbers)*
7. *M. A. Armstrong, Basic Topology, Springer.*

MATH-CCT-203: Functional Analysis **Marks: 50+10=60** **Credit: 2.4**

Total Lectures: 50 Hrs.

Metric space: Definitions and examples; spaces like l_p, l_∞, $C[a, b]$, Holder and Minkowski inequalities for sums, completeness of metric spaces, isometric mapping, isometric spaces, completion of metric spaces, Baire category theorem, Banach's fixed point theorem and its applications.
Normed spaces, Banach spaces and fundamental theorems: Normed linear spaces, Banach spaces and examples, quotient space of normed spaces and its completeness, Schauder basis, normed space and its completion, equivalent norms, finite dimensional normed linear spaces and compactness, Riesz Lemma, bounded linear operators, its equivalence with continuity, normed linear spaces of bounded linear operators, linear functional, bounded linear functional, dual space and second dual space, canonical embedding, algebraic reflexivity, strong and weak convergence, uniform boundedness theorem and some of its consequences, open mapping theorem, closed graph theorem, Hahn-Banach theorem for real linear spaces, complex linear spaces and some of its consequences.
Inner product spaces and Hilbert spaces: Inner product spaces, Hilbert spaces and examples, Cauchy-Schwarz inequality, triangle inequality, parallelogram law, Inner product spaces and its completion, orthonormal sets and sequences, orthogonal complements and direct sums, Bessel's inequality, Gram-Schmidt orthonormalization process, complete

orthonormal sets and Parseval's identity, Riesz representation theorem, Sesquilinear form, separable and non-separable Hilbert space, reflexivity of Hilbert spaces.

Operators: Adjoint of an operator on a Hilbert space, Self-adjoint operators, normal and unitary operators, convergence of sequences of operators and functionals, positive operators, projection operators, compact operators.

Recommended Books:

1. *K. Kreyszig , Introductory Functional Analysis with Applications, John Wiley &Sons New York, 1978.*
2. *B. K. Lahiri, Elements of Functional Analysis, The World Press Pvt. Ltd. Calcutta, 1994.*
3. *G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Co. New York, 1963.*
4. *J. B. Conway, A course in functional analysis, Springer-Verlag, New York 1990*

MATH-CCT-204: Classical Mechanics and Calculus of Variation

Total Lectures: 50 Hrs. Marks: 50+10=60 Credit: 2.4

Classical Mechanics: Introduction, generalized coordinates, degrees of freedom, virtual work, D'Alembert's principle, unilateral and bilateral constraints, holonomic and nonholonomic systems, scleronomic and rheonomic systems, Lagrange's equations for holonomic systems, Lagrange's equation for impulsive forces and for systems involving dissipative forces, conservation theorems, Hamilton's variables, Hamilton's canonical equation, homogeneity of space and time, conservation principles, Noethers theorem, cyclic coordinates, generalized momentum, Routh's equation, Hamilton's principle and principle of least action, canonical transformation with different generating functions, Lagrange and Poisson brackets and their properties, Hamilton-Jacobi equations, Poisson's identity, Jacobi-Poisson theorem.

Calculus of Variation: Introduction, Euler's equation, different forms of Euler's equations, solutions of Euler's equation, geometrical problems, geodesics, minimum surface of revolution, isoperimetric problems, Brachistochrone problem, variational problems involving several unknown functions, functionals dependent on higher order derivatives, variational problems involving several independent variables, constraints and Lagrange's multipliers, moving boundaries, sufficient conditions for extremum, variational formulation of Boundary Value Problem, minimum of quadratic functional, approximate methods, Rayleigh-Ritz method, Galerkin's method, weighted-residual methods, collocation methods, variational methods for time dependent problems.

Recommended Books:

1. *H. Goldstein, Classical Mechanics, Pearson New International edition, Third edition, (2014).*
2. *P. S. Jog, N. C. Rana, Classical Mechanics, McGraw-Hill, First edition, (2001).*
3. *S. L. Gupta, V. Kumar, H. V. Sharma, Classical Mechanics, Pragati Prakashan, (2010).*
4. *R. G. Takwale, P.S. Puranik, Introduction to Classical Mechanics, Tata McGraw-Hill, (1979).*
5. *A. S. Gupta, Calculus of Variations with Applications, Prentice Hall of India, (2015).*
6. *L. D. Elsgole, Calculus of Variation, Dover Publication, Inc. New York, (2007).*
7. *I. M. Gelfand, S. V. Fomin, Calculus of Variation, Dover books on Mathematics (2000).*

MATH-CCT-205: Integral Equations and Integral Transforms
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

Integral Equations: Classifications, successive approximations, separable kernels, Fredholm alternative, Hilbert-Schmidt theory of symmetric kernels, construction of Green's function, convoluted kernels, Abel's equations and solutions.
Integral Transforms: Laplace and Fourier transforms, applications to Boundary Value Problems, Mellin & Hankel transformation, inversion formulae, Bromwich Integral, convolutions and applications, distributions and their transforms, applications to Wave, Heat and Laplace equations.

Recommended Books:

1. *I. N. Sneddon, The Use of Integral Transform, Tata-McGrawHill, (1974).*
2. *R. Churchil, J. Brown, Fourier Series and Boundary Value Problems, McGraw- Hill, (1978).*
3. *F. G. Tricomi, Integral Equations, Dover Publications, (1985).*
4. *A. M. Wazwaz, Linear and Nonlinear Integral Equations, Springer, (2011).*
5. *P. P. G. Dyke, An introduction to Laplace Transforms and Fourier Series, Springer, (2014).*
6. *M. G. Spiegel, Laplace Transforms (Schaum's OuTlines series), McGraw-Hill Education, (2002).*
7. *J. L. Schiff, The Laplace Transform, Springer, (2013).*
8. *L, Debnath, D, Bhatta, Integral Transforms and Their Applications, CRC Press, (2016).*

MATH-CCT-206: Partial Differential Equations
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

General solution and complete integral of a partial differential equation, singular solution, integral surface passing through a curve and circumscribing a surface.
First Order PDE: Formation and solution of PDE, integral surfaces, Cauchy's method of characteristic, orthogonal surfaces, First order non-linear PDE, compatible system, Charpit's method, classification and canonical forms of PDE.
Second Order PDE: Linear partial differential equations of second and higher order, classification of Second order PDE, Monge's method. reduction to canonical form, solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables, linear PDE with variable coefficients, canonical forms, canonical transformation of linear second order PDE.
Elliptic Differential Equations: Derivation of Laplace and Poisson equation, Boundary Value Problem, Harmonic functions, characterization of harmonic function by their mean value property, method of separation of variables for the solutions of Laplace's equations, Dirichlet Problem and Neumann Problem for a rectangle, interior and exterior Dirichlet problems for a circle and a semi-circle, interior Neumann problem for a circle, solution of Laplace equation in cylindrical and spherical coordinates, steady-state heat flow equation Problems, Poisson's general solution, examples.
Parabolic Differential Equations: Formation and solution of diffusion equation, initial and boundary conditions, separation of variables method, solution of heat equation under Dirichlet and Neumann condition, heat conduction problem for an infinite rod, heat conduction in a finite rod, solution of parabolic equation under non-homogeneous boundary condition, solution of diffusion equation in cylindrical and spherical coordinates, examples.

Hyperbolic Differential Equations: Formation and solution of one-dimensional wave equation, canonical reduction, initial value problem, D'Alembert's solution, vibrating string, forced vibration, method of separation of variables, initial value problem and boundary value problem for two- dimensional wave equation, initial value problem for a non-homogeneous wave equation, higher-dimensional wave equations, periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, uniqueness of the solution for the wave equation, Duhamel's Principle, examples.
Generalized function: Elementary properties of generalized functions, addition, multiplication, transformation of variables, generalized function as the limit of a sequence of good functions, even and odd generalized functions, differentiation and integration of generalized functions, simple examples, ordinary function as generalized function, Anti-derivative, regularization of divergent integral with simple example, Fourier transform of generalized function, examples, convergence of a sequence of generalized functions with examples, Laplace transform of generalized function.

Recommended Books:

1. *I. N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, (1986).*
2. *L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, (1998).*
3. *R. C. McOwen, Partial Differential Equations (Pearson Edu.), (2003).*
4. *T. Amarnath, Partial Differential Equation, Alpha Science International, (2003).*
5. *P. P. Prasad, R. Ravichandan – Partial Differential Equations, Wiley Eastern, (1985).*
6. *M. G. Smith, Introduction to the Theory of Partial Differential Equations, Van Nostrand, (1967)*
7. *F. H. Miller, Partial Differential Equations, Wiley, New York, (1941).*
8. *R. Churchil, J. Brown, Fourier Series & Boundary Value Problems, McGraw-Hill, New York, (1987).*
9. *G. Greenspan, Introduction to Partial Differential Equations, Dover Publications, (2012).*
10. *D. S. Jones, Generalized Functions, Cambridge University Press, (1982).*

MATH-CCT-301: Differential Geometry and Its applications

Total Lectures: 50 Hrs. **Marks: 50+10=60** **Credit: 2.4**

Differential Geometry:
Vectors, one-forms (dual vectors), manifolds, differentiable manifolds and tensors, metric tensor on manifolds, differential forms: Hodge duality, differential structure on manifold: Lie derivative, Exterior differentiation, affine connection, Covariant differentiation and Intrinsic differentiation; Absolute differentiation: connection forms, Lie bracket, Holonomic and Anholonomic bases.
Geometry of Space Curves: Serret-Frenet formulae, Equation of Straight lines, Helix, Bertrand curve.
Geometry of Surfaces: Regular surfaces, differential functions on surfaces, the tangent plane and the differential maps between regular surfaces, the first fundamental form, normal fields and orientability, Gauss map, the second fundamental form, normal and principal curvatures, Gaussian and mean curvatures.
Geodesics and Curvature: Autoparallel curves and geodesics, Exponential map, Parallel transport, geodesic coordinates, curvature, Riemann curvature tensor and Theorem of Egregium, geodesic curvature, Ricci tensor, Ricci scalar, Curvature 2-forms, geodesic deviation, Bianchi identities.

Applications of Differential Geometry (Special Theory of Relativity):
Inertial frame of references, postulates of special theory of relativity, Lorentz transformation from geometric point of view; consequences of Lorentz transformation: length contraction, time dilation; laws of composition of velocities, relativistic mechanics: world line, world events and world region, proper time as an invariant arc length, relativistic mass and energy and momentum, equivalence of mass and energy; the space-time geometry: Minkowskian 4D space-time, four-vector, time-like, light-like and space-like vectors, light cone, time-like, space-like, light-like intervals, null cone.

Recommended Books:

1. *C. Bar, Elementary Differential Geometry, Cambridge University Press, 2011.*
2. *K. Tapp, Differential Geometry of Curves and Surfaces, Springer,2016.*
3. *A. Pressley, Elementary Differential Geometry, Springer, 2010.*
4. *M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice- Hall, Inc., Upper Saddle River, New Jersey 07458, 1976.*
5. *S. Chakraborty, A Treatise on Differential Geometry and its role in Relativity Theory, Book ref: arXiv:1908.10681[gr-qc] (2019).*
6. *R. Resnick, Introduction to Special Relativity, Wiley Student Edition, 1968.*
7. *J. V. Narlikar, An Introduction to Relativity, Cambridge University Press, 2010.*
8. *L. Ryder, Introduction to General Relativity, Cambridge University Press, 2009.*
9. *B. F. Schutz, A First Course in GENERAL RELATIVITY, Cambridge University Press, 2009.*
10. *S. M. Carroll, Space-time and Geometry, Cambridge University Press 2019.*

MATH-CCT-302: Field Extension and Galois Theory Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Field Extension: Fields, Algebraic and transcendental extensions, finite extensions, fundamental theorem of general algebra (Krönekar Theorem), Splitting fields, algebraically closed fields, Separable and Purely inseparable extensions, Perfect fields, Normal extensions, Finite fields.
Galois Theory: Galois Extensions, Galois Groups, Fundamental theorem of Galois theory, Cyclotomic extensions, Solvability by radicals, Geometric constructions by straightedge and compass only.

Recommended Books:

1. *D. S. Malik, J.M. Mordeson, and M.K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill, International Edition, (1997).*
2. *D. S. Dummit and R.M. Foote, Abstract Algebra (3e), John Wiley and Sons (Asian reprint).*
3. *S. Lang, Algebra, 3rd edition., Springer (Indian reprint 2004).*
4. *I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. (1975).*
5. *J. J. Rotman, Advanced Modern Algebra, Springer (Indian reprint 2016).*
6. *I. S. Luther and I.B.S. Passi, Algebra, Vol. IV - Field Theory, Narosa Publishing House.*

MATH-CCT-401: Graph Theory, Algorithms, and Combinatorics

Total Lectures: 50 Hrs. Marks: 50+10=60 Credit: 2.4

Graph Theory and Algorithms:
Preliminaries: Undirected and directed graphs, subgraphs, algebra on graphs, matrix representation, bipartite graphs, line graphs, chordal graphs, etc.

Connected Graphs and Shortest Paths: Walks, Trails, Paths, Distance, Diameter, Eccentricity, Blocks, Articulation points, Bridges, Cut-vertices, Cut-edges, biconnected and strongly connected components, Weighted graphs,

Shortest path algorithms for undirected and directed graphs: BFS, DFS, single source shortest path (S3P), all pair shortest path (APSP).

Euler and Hamiltonian graphs: Characterization, Konigsberg bridge problem, Petersen graph, Fleury's algorithm, Chinese-postman-problem, orientation in a directed graph, Eulerian directed graphs, Hamilton directed graphs, tournaments, maximum flow, minimum cost

Trees: Characterization and basic properties, binary trees, height of a binary tree, binary search tree, height balanced tree, red-black tree, B-tree, minimum spanning trees, Kruskal algorithm.

Tree Traversal: DFS, BFS, pre-order, in-order, post-order.

Vertex and Edge colorings: Chromatic number and Chromatic index, cliques, greedy coloring algorithm, coloring of chordal graphs, Brook's theorem, Gupta-Vizing theorem, Class-1 and class-2 graphs, equitable edge coloring.

Independent Sets, Coverings, Matching: Basic equations, matching in bipartite graphs, perfect matching, greedy and approximation algorithms.

Planar Graphs: Basic concepts, Euler's formula, polyhedrons and planar graphs, characterizations, planarity testing, 5-color-theorem.

Combinatorics: Poset, Graphs of relations, Hasse diagram, matrices of relations, closure operations on relations - Warshall's algorithm, the basic problem of combinatorics-counting, generalized permutations and combinations, Pigeonhole principle, binomial coefficients and identities, recurrence relations, applications of recurrence relations, solving linear recurrence relations, generating functions, definition with examples, solving recurrence relations using generating functions, exponential generating functions, difference equations, principle of inclusion–exclusion, applications of inclusion–exclusion, modeling with recurrence relations with examples of Fibonacci numbers and the tower of Hanoi problem.

Recommended Books:

1. *J. P. Tremblay, R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill Book Co. (1997).*
2. *N. Deo, Graph Theory with Applications to Engineering and Computer Sciences, Prentice Hall of India.*
3. *D. B. West, Introduction to Graph Theory, Prentice-Hall of India/Pearson, (2009).*
4. *J. A. Bondy, U. S. R. Murty: Graph Theory and Applications, New-Holland, New York, (1976).*
5. *R. Balakrishnan, K. Ranganathan, A Textbook of Graph Theory, 2nd edition, Springer, (2012).*
6. *R. A. Brualdi, Introductory Combinatorics, Prentice Hall (India).*
7. *M. O. Albertson, J. P. Hutchinson, Discrete mathematics with algorithms, John Wiley and Sons, (1988).*
8. *J. H. Van Lint, R. M. Wilson, A course in Combinatorics, Cambridge University Press, (1992).*
9. *J. L. Mott, A. Kandel, T. P. Baker, Discrete mathematics for Computer Scientists and Mathematicians, 2nd edition, Prentice Hall (India), (1999).*
10. *B. Kolman, R. C. Busby, S. Ross, Discrete mathematical structures, 3rd edition, Prentice Hall (India), (1999).*

MATH-CCP-402: Numerical Problem Solving by Computer Programming (PRACTICAL) Total Lectures: 50 Hrs. Marks: 25+35=60 Credit: 2.4

Group-A

C Programming: An overview of computer programming languages, modular programming and program development cycle, character set, keywords and identifiers, variables and Constants.

Fundamental Data Types: Int, short, long; float, double; char; type conversion and casting; Operators and Expressions: arithmetic operators, relational operators, logical operators, assignment operators, increment and decrement operators, bitwise manipulation operators, size of operator, conditional operator; operator precedence and associativity; void data type.

Conditional Branching: if, if-else, switch; Looping and nested looping, for, while, do-while; break and continue, goto, infinite loops, header file and include directive, macro substitution and conditional compilation, scanf, printf and various format specifiers, standard C library functions, declaring, initializing and using arrays in programs; arrays and memory; one dimensional and multidimensional arrays; character arrays and strings, pointer arithmetic, accessing array elements through pointers, arrays of pointers, pointers to pointers, sorting algorithms, passing arguments to a function, declaring and calling a function; pointers to functions, passing arrays as function arguments, recursion, main() function. opening and closing a file, reading from a file and writing to a file, random access and error handling.

Group- B

Solving Numerical Problems using C – Programming

1) Interpolation: Newtown forward, Newtown backward, Stirling, Lagrange, Divided Differences.
2) Differentiation: using interpolated polynomials.
3) Integration: Trapezoidal Method, Simpson Method, Romberg Method, Gauss Quadrature Method.
4) Matrix inversion: Gauss Jordan method.
5) Largest eigenvalue and corresponding eigen vector of a square matrix: Power method.
6) System of Linear equation: Gauss Elimination method, Gauss-Jacobi method Gauss-Seidal method.
7) O.D.E.: Runge-Kutta method, Milne’s method, Adams method.
8) P.D.E. (finite difference method): Laplace, Parabolic, and Hyperbolic.

Recommended Books:

1. *B. Gottfried, Programming with C, Tata McGraw-Hill, (2002).*
2. *E. Balagurusamy, Programming in ANSI C, Tata Mcgraw Hill, (2002).*
3. *B. W. Kernighan, D. M. Ritchie, The C Programme Language, 2nd Edition (ANSI features), Prentice Hall, (1989).*
4. *Y. P. Kanithkar, Let Us C- BPB Publication, (2002).*
5. *M. K. Jain, Numerical Methods For Scientific And Engineering Computation, Wiley, (2003).*
6. *G. D. Smith, Numerical Solution of Partial Differential Equations by Finite Difference Methods, Clarendon Press,(1985).*
7. *K. R. Venugopal, S. R. Prasad, Programming with C, Tata-McGraw Hill.*

MATH-ECT-303(M1): Measurability and Integration in Abstract Spaces
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

Construction of measure by means of outer measure, regular outer measure and metric outer measure, basic properties, construction of outer measure.
Integration on measure spaces, integration of simple functions, integration of bounded functions on sets of finite measure, Lebesgue integral of non-negative functions, monotone convergence theorem, Lebesgue integral of measurable functions, convergence theorems, translation and linear transformation of the Lebesgue integral on $\mathbb{R}$.

<u>Recommended Books:</u>

1. *J. Yeh, Lectures on Real Analysis, World Scientific, (2000).*
2. *H. L. Royden, Real analysis, Macmillan Publishing Co., Inc. 4th Edition, (1993).*
3. *S. K. Berberian. Measure and integration. Chelsea Publishing Company, NY, (1965).*
4. *G. D. Barra, Measure Theory and integration, Wiley Eastern Ltd, (1981).*
5. *R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, (1966).*
6. *I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi (1997).*

MATH-ECT-303(M2): Topological Groups **Marks: 50+10=60 Credit: 2.4**
Total Lectures: 50 Hrs.

Definition and examples of topological groups, Topologies generated by characters, Pseudonorms and invariant pseudometrics in a group, Function spaces as topological groups, Transformation groups, Subgroups and direct product of topological groups, Quotients of topological groups, initial and final topologies, Separation axioms, closed subgroups, Metrizability of topological groups, Connectedness in topological groups, Group topologies determined by sequences. The Zariski topology and the Markov topology, The Markov topology of the symmetric group, Existence of Hausdorff group topologies, Extension of group topologies, Cardinal invariants of topological groups, Completeness and completion of topological groups, Compactness and local compactness in topological groups: examples, specific properties of compactness and local compactness, general properties (the open mapping theorem, completeness, etc.), compactness vs connectedness, Properties of $\mathbb{R}^n$and its subgroups, The closed subgroup of $\mathbb{R}^n$, elementary LCA groups and Kronecker's theorem, on the structure of compactly generated locally compact abelian groups.

<u>Recommended Books:</u>

1. *D. Dikranjan, Introduction to topological groups.*
2. *S. Warner, Topological Rings, Elsevier Science Publishers.*
3. *I. F. Wilde, Topological Vector Spaces (Lecture notes).*

MATH-ECT-303(M3): Algebraic Topology **Marks: 50+10=60 Credit: 2.4**
Total Lectures: 50 Hrs.

Homotopy Theory : Homotopy of maps, multiplication of paths, Fundamental Groups, , induced homomorphisms, Fundamental groups of Circle,Sphere and some surfaces. covering spaces, lifting theorems, the universal covering space, Seifert-van Kampen theorem, applications. Geometrical construction of group structure on circle (in fact on any conic), Separation Theorem in the plane, Classification of surfaces.
Simplicial Homology: Simplicial complexes, chain complexes, definitions of the simplicial homology groups, properties of homology groups, applications.

Recommended Books:

1. *M. A. Armstrong, Basic Topology, Springer (India), (2004).*
2. *A. Hatcher, Algebraic Topology, Cambridge University Press, (2002).*
3. *C. A. Kosniowski, First Course in Algebraic Topology, Cambridge Univ. Press, (1980).*
4. *F. H. Croom, Basic Concepts of Algebraic Topology, Springer-Verlag, (1978).*
5. *S. Deo, Algebraic Topology-A Primer, Hindustan Book Agency.*
6. *J. R. Munkres, Topology, PHI.*
7. *A. R. Shastri, Basic Algebraic Topology, CRC Press Book.*

MATH-ECT-303(M4): Elementary Number Theory Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Linear congruences, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.
Arithmetic functions, multiplicative functions, Mobius inversion formula, Euler's theorem, structure of units modulo *n*, primitive roots, quadratic residues, law of quadratic reciprocity.
Representation of integers as sum of squares, Fermat's two square theorem, Lagrange's four-square theorem.
Dirichlet product of Arithmetic functions, Characters of a finite abelian group. Introduction to Riemann Zeta function and infinitude of primes, infinitude of primes in specific arithmetic progressions, Dirichlet's theorem (without proof).
Algebraic number fields and the ring of integers. Trace and norm of an element in a field, units and primes, factorization, quadratic and cyclotomic fields.

Recommended Books:

1. *D. M. Burton, Elementary Number Theory, University of New Hampshire.*
2. *T. M. Apostol, Introduction to Analytic number theory, UTM, Springer, (1976).*
3. *G. H. Hardy, E.M. Wrigh,. An Introduction to the Theory of Numbers (6th ed, Oxford University Press, (2008).*
4. *D. A. Marcus, Number Fields, Universitext, Springer-Verlag, (1977).*
5. *G. J. Janusz, Algebraic Number Fields, Graduate Studies in Mathematics 7, American Mathematical Society, (1996).*
6. *S. Lang, "Algebraic Number Theory", Graduate Texts in Mathematics 110, Springer-Verlag, (1994).*
7. *J. W. S. Cassel, A. Frolich, Algebraic number theory, Cambridge.*
8. *K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, (1968).*
9. *H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, (2004).*
10. *R. A. Mollin, Algebraic Number Theory, Chapman & Hall/CRC.*

MATH-ECT-303(M5): Advanced Complex Analysis-I Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

The functions M(r) and A(r). Theorem of Borel and Caratheodary, Convex function and Hadamard three-circle theorem.
Convergence and Compactness in the space of analytic function, Riemann Mapping theorem.
Entire functions, growth of an entire function, order and type and their representations in terms of the Taylors coefficients, distribution of zeros, Picards's first theorem. Weierstrass factorization theorem, the exponent of convergence of zeros. Canonical product, Hadamard's factorization theorem, Borel's theorems, Picard's second theorem.

Recommended Books:

1. *A. I. Markusevich , Theory of Functions of a Complex Variables, Vol. I & II, Printice-Hall, (1965).*
2. *A. S. B. Holland, Introduction to the theory of entire function, Academic Press New York and London, (1973).*
3. *J. B. Conway, Functions of One Complex Variable, Narosa Publishing House, New Delhi, 2nd Edn, (1997).*
4. *L. V. Ahlfors, Complex Analysis, McGraw-Hill, 3rd Edn. (1979).*
5. *R. P. Boas, Entire Functions, Academic Press, (1954).*
6. *H. Cartan, Theory of Analytic Functions, Dover Publication, (1995).*
7. *T. W. Gamelin, Complex Analysis, Springer, New York, (2001).*

MATH-ECT-303(M6): Advanced Functional Analysis-I Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Approximation in Normed Spaces: Best approximation. Existence theorem of best approximations. Strict convexity. Hilbert space is strictly convex. The space C[a,b] with sup-norm is not a strictly convex. Uniqueness theorem of best approximation. Uniform approximation. Extremal point. Haar condition. Haar uniqueness theorem. Chebyshev polynomials. Approximation in Hilbert space. Spline approximation.
Spectral Theory of Linear Operators in Normed Spaces: Spectral theory in finite dimensional normed spaces. Eigenvalue of an operator. Regular value of an operator. Resolvent set of an operator. Spectrum of an operator. Spectral properties of bounded linear operators. Spectral radius. Spectral mapping theorem for polynomials. Banach Algebra. Compact linear operators on normed spaces and their spectrum. Spectral properties of compact linear operators on normed spaces. Spectral theory of bounded self-adjoint linear operators. Spectral properties of bounded self-adjoint linear operators. Spectral family of a bounded self-adjoint linear operator. Spectral representation of bounded self-adjoint linear operators. Extension of the spectral theorem to continuous functions. Unbounded linear operators in Hilbert space. Hellinger-Toeplitz theorem of boundedness. Spectral properties of self-adjoint linear operator.

Recommended Books:

1. *C. Goffman, G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, (1987).*
2. *K. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons New York, (1978).*
3. *B. K. Lahiri, Elements of Functional Analysis, The World Press Pvt. Ltd. Calcutta, (1994).*
4. *G. F. Simmons, Introduction to Topology and Modern Analysis, Tat McGraw-Hill Edition (2004).*
5. *K. Yosida, Functional Analysis, 3rd edition Springer - Verlag, New York, (1971).*

MATH-ECT-303(M7): Theory of Approximation Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Concept of Best Approximation in a Normed Linear Space, Existence of the Best Approximation, Uniqueness Problem, Convexity-Uniform Convexity, Strict Convexity and their relations.
Continuity of the best Approximation Operator. The Weierstrass Theorem, Bernstein Polynomials, Korovin Theorem, Algebraic and Trigonometric polynomials of the best

Approximation. Lipschitz class, Modulus of Continuity, Integral Modulus of Continuity and their properties.
Bernstein's Inequality. Jacson's Theorems and their Converse Theorems. Approximation by means of Fourier Series.
Positive Linear Operators, Monotone Operators, Simultaneous Approximation, L^p approximation. Approximation of analytic Functions.

Recommended Books:

1. *H. M. Mhaskar, D. V. Pai, Fundamentals of Approximation Theory, Narosa Publishing House.*
2. *A. F. Timan, Theory of Approximation of Functions of a Real Variable, Dover Publication Inc.*
3. *E. W. Cheney, Introduction to Approximation Theory, AMS Chelsea Publishing Co.*
4. *G. G. Lorentz, Bernstein Polynomials, Chelsea Publishing Co.*
5. *I. P. Natanson, Constructive Function Theory Volume-I, Fredrick Ungar Publishing Co.*

MATH-ECT-303(M8): p-adic Analysis Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Completion from $\mathbb{Q}$ to $\mathbb{R}$, normed fields, non-Archimedean norm; basic properties, construction of the completion of a normed field, the field of p-adic numbers $\mathbb{Q}_p$, arithmetical operations in $\mathbb{Q}_p$, p-adic integers, p-adic rational numbers; a necessary and sufficient condition for x in $\mathbb{Q}_p$ to be rational, uncountability of the set of p-adic integers, Hensel's lemma and congruences, p-adic norm on $\mathbb{Q}$; Ostrowski's theorem.
Topology of $\mathbb{Q}_p$, spheres and balls in $\mathbb{Q}_p$, countability of set of all balls in $\mathbb{Q}_p$, local compactness and totally disconnectedness of $\mathbb{Q}_p$.
Cantor sets, the set $\mathbb{Z}_2$ of 2-adic integers is homeomorphic to Cantor set C, basic convergence properties of sequences and series in $\mathbb{Q}_p$, p-adic power series; its convergence.

Recommended Books:

1. *S. Katok, p-adic analysis compared with real, American Mathematical Society (2007).*
2. *F. Q. Gouvˆea, p-adic Numbers: An Introduction, 2nd edition, Springer-Verlag (1997).*
3. *R. W. Vallin, The Elements of Cantor Sets: With Applications, Wiley.*
4. *A. M. Robert, A course in p-adic analysis, Springer-Verlag, (2000).*
5. *G. Bachman, Introduction to p-adic numbers and valuation theory, Academic Press (1964).*

MATH-ECT-304(N1): Advanced Numerical Analysis Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Revision of numerical interpolation, differentiation and integration, least squares approximation, orthogonal polynomials, Gram-Schmidt orthogonalization, Chebyshev polynomials, Lagrange's interpolation, divided differences and their properties, generalized Newton's divided difference interpolation formula, piecewise polynomial interpolation, Hermite interpolation formula, cubic spline interpolation and its convergence, errors in Newton-Cotes quadrature formula, Gauss-Legendre and Gauss-Chebyshev quadrature formulae, Romberg integration, errors, vector and matrix norms, spectral radius, conditional number.
Direct and Iterative Method for Solving System of Linear Equations: Gauss Jordan, LU decomposition, Cholesky's decomposition, Gauss Jacobi, Gauss Seidel method, successive over-relaxation method (SOR).

Computation of eigenvalues and eigenvectors of matrices by iteration methods: Power method, Jacobi method etc, Gauss quadrature formula and applications.
Numerical solutions of initial value problems using generalized Runge Kutta methods (implicit and explicit).
Finite difference methods: Forward difference, backward difference, and forward in time centred in space (FTCS), backward in time centred in space (BTCS), Crank-Nicolson method.
General single step and multistep methods and some examples, shooting method and Galerkin method.

Recommended Books:

1. *M. K. Jain, S. R. K. Iyenger and R. K. Jain Numerical Methods for Scientific and Engineering Computations, New Age International Publication (P) Ltd, New Age International (P) Limited, (2003).*
2. *K. E Atkinson, An Introduction to Numerical Analysis (Second edition), John Wiley and Sons, (2008).*
3. *R. W. Hamming, Numerical Methods for Scientists and Engineers (Dover Books on Mathematics) 2nd Revised ed. Edition, Dover publications, (1986).*
4. *J. F. Epperson, An introduction to numerical methods and analysis (second edition), John Wiley and Sons, (2013).*
5. *D. Kincaid, W. Cheney, Numerical Analysis: Mathematics of Scientific Computing (The Sally Series; Pure and Applied Undergraduate Texts, Vol. 2), American Mathematical Society; 3rd Revised Edition, (2002).*

MATH-ECT-304(N2): Continuum Mechanics Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Introduction: Mathematical preliminaries: vector spaces, index notation, second order tensors: skew symmetric, orthogonal and symmetric tensors, invariants of second-order tensors, eigenvalue problem, positive definiteness and polar decomposition theorem, isotropic functions, and higher-order tensors; directional derivative, Frechet derivative, gradient, divergence, curl, and integral theorems, transport theorem, configurations of a body, displacement, velocity, motion, deformation gradient, rotation, stretch, strain, strain rate, spin tensor, assumption of small deformation and small strain.
Lagrangian and Eulerian description, deformation gradient, strain tensor, stretch tensors, area and volume transformation, material and spatial derivative, rate of deformation and spin tensors, Reynold’s transport theorem, vorticity and circulation.
Balance Laws: Balances of mass, linear momentum and angular momentum, contact forces and the concept of stress, balance of energy and Clausius-Duhem inequality, constitutive relations, frame indifference, material symmetry, kinematic constraints (incompressibility), thermodynamical restrictions.
Viscous Fluid: Constitutive relations, incompressible fluids, non-Newtonian fluid, boundary value problem.
Finite Elasticity: Hyper-elasticity, isotropy, linear elasticity, simple constitutive relations, boundary value problem.

Recommended Books:

1. *J. N. Reddy, An Introduction to Continuum Mechanics (Second edition), Cambridge University Press.*

2. *D. Rubin, E. Krempl, W. Michael Lai, Introduction to Continuum Mechanics (Third edition), Pergamon press, (1993).*
3. *Peter Chadwick, Continuum Mechanics: Concise Theory and Problems (Second Edition), Dover Publication Inc. (1999).*
4. *J. W. Rudnicki, Fundamentals of Continuum Mechanics, John Wiley & Sons, (2014).*
5. *A. J. M. Spencer, Continuum Mechanics, Dover Publication Inc. (2004).*

MATH-ECT-304(N3): Computational Partial Differential Equations
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

Introduction: Classifications of PDE, finite Difference Methods (forward, backward and central), convergence and consistency, CFL number and Fourier and Von Neumann stability analysis for Finite Difference Method, theory of well-posed IVPs scalar and systems, convergence estimates for smooth and non-smooth initial conditions, convergence estimate for parabolic differential equations, Lax-Richmyer equivalence theorem, well-posed and stable initial BVP, matrix method for stability.
Parabolic PDE and Examples: Solution for one dimensional equation, *explicit and various implicit schemes:* Backward in time centered in space (BTCS), Forward in time centered in space (BTCS), Crank-Nicolson scheme (CNS) etc., tri-diagonal system, discussion on compatibility, stability and convergence of above schemes, extension to 2d parabolic equations examples.
Elliptic PDE and Examples: Solution of Laplace, Poisson etc. PDE in Cartesian and Polar system, ADI and SOR schemes etc., Laplace equation using standard five point formula and diagonal five point formula, methods for solving diagonal systems, treatment of irregular boundaries.
Hyperbolic PDE and Examples: Wave equation, Finite difference explicit and implicit schemes-Upwind scheme, Lax-Wendroff schemes, MacCormak schemes, stability analysis, method of characteristics and their significance.
Introduction to Finite Element Method, Finite Volume Method.

Recommended Books:

1. *M. K. Jain, S.R.K. Iyenger, R.K. Jain, Computational Methods for Partial Differential Equations (Second edition), New Age International Publication (P) Ltd, (2016).*
2. *K. W. Mortons, D. F. Mayers, Numerical Solution of Partial Differential Equations (Second edition), Cambridge University Press.*
3. *V. Ruas, Numerical Methods for Partial Differential Equations: An Introduction, Wiley, (2016).*
4. *S. Mazumder, Numerical Methods for Partial Differential Equations 1st Edition, Academic Press, USA.*

MATH-ECT-304(N4): Dynamical System **Marks: 50+10=60 Credit: 2.4**
Total Lectures: 50 Hrs.

Discrete and continuous time dynamical systems, flows and maps, phase space, orbits, fixed points, periodic points and their stability, attractors and repellors, logistic map, tent map, Baker's map, graphical analysis of orbits of one-dimensional maps, hyperbolicity.
General solution of continuous time linear systems, phase space diagrams, fixed point analysis, stable and unstable nodes, saddle point, stable and unstable foci, centre.

Lyapunov and asymptotic stability, local and global stability, Hartmann-Grobman theorem, Lyapunov theorem on stability, Lyapunov functions, periodic orbits, limit cycles, attracting and invariant set, Poincare-Bendixson theorem, Poincare map.
Sensitive dependence on initial conditions (SDIC), topological transitivity, topological mixing, topological conjugacy and semi-conjugacy for maps, chaos, chaotic orbits, Lyapunov exponents, invariant measure.
Ergodic maps, invariant measure for logistic maps.
Symbolic dynamics, shift map, properties of logistic map, Cantor set, Cantor set structure of logistic map, topological conjugacy of logistic and shift map, chaotic behaviour of logistic map.
Bifurcation Theory: Saddle-node bifurcation, Pitchfork bifurcation, period doubling bifurcation, period doubling route to chaos, Hopf bifurcation, Sarkovskii's theorem, period-three implies chaos for 1-D maps, two-dimensional maps, Toral automorphism, problems.

Recommended Books:

1. *L. Perko, Differential Equations and Dynamical Systems, Springer, (2001).*
2. *S, Wiggins, Introduction to Apllied Non-Linear Dynamical Systems and Chaos, Springer New York, (1990).*
3. *S. H. Strogatz, Nonlinear Dynamics and Chaos, CRC Press, (2018).*
4. *M. W. Hirsch and S. Smale, Differential Equations, Dynamical Systems, AcademicPress (1974).*
5. *S. L. Ross, Differential Equations, 3rd Edn., Wiley India, (1984).*
6. *G. C. Layek, An Introduction to Dynamical Systems, Springer, (2015).*
7. *M. W. Hirsch, S. Smale, and R. L. Devaney, Differential Equations, Dynamical Systems and an Introduction to Chaos, Academic Press, (2012).*

MATH-ECT-304(N5): Fluid Mechanics **Marks: 50+10=60 Credit: 2.4**

Total Lectures: 50 Hrs.

Kinematics of Fluids in Motion: Continuum hypothesis, Lagrangian and Eularian description, stream lines, path lines, streak lines, vortex lines, velocity potential, vorticity vector, equation of continuity.
Equations of Motion for Inviscid Flow: Pressure at a point in a moving fluid, conditions at boundary, Euler's equations of motion, Bernoulli's equation, potential flows, *two-dimensional flows:* Sources, sinks, doublets, images in rigid infinite plane and in solid sphere, Weiss' sphere theorem, Butler's sphere theorem, flows involving axial symmetry, Stokes stream function, complex velocity potential for two dimensional irrotational-incompressible flows, two-dimensional image systems, Milne-Thomson circle theorem and its applications, Blasius theorem.
Viscous Flows: Stress analysis in fluid motion, relations between stress and rate of strain, constitutive equations, derivation of Navier-Stokes equations, *exact solutions of Navier-Stokes equations:* plane Poiseuille flow and Couette flow, Hagen-Poiseuille flow, flow between two concentric rotating cylinders, Stokes first and second problem, viscous flow past a sphere, Reynolds number, Prandtl's boundary layer theory, Karman's integral equation, similarity solution, boundary layer for an axially symmetric flow, laminar flow with adverse pressure gradient and separation. Slow viscous flow: Stokes and Oseen's approximation, theory of hydrodynamic lubrication.
Introduction to Hydrodynamic Stability: Linear stability of plane Poiseuille flow, Orr-Sommerfeld equation, description of turbulent flow, velocity correlations, Reynolds stresses,

Prandtl's mixing length theory, Karman's velocity defect law, universal velocity distribution, concepts of closure model, eddy viscosity models of turbulence, zero equation, one equation and two-equation models.

Recommended Books:

1. *F. M. White, Fluid Mechanics, McGraw-Hill Higher Education, 8th edition.*
2. *I. Kohen, P. K. Kundu, Fluid Mechanics, Elsevier, 3rd edition.*
3. *F. M. White, Viscous Fluid Flow, McGraw-Hill Higher Education, 3rd edition*
4. *G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press.*
5. *S. K. Som, G. Biswas and S. Chakraborty, Introduction to Fluid Mechanics and Fluid Machines, Tata-McGraw-Hill, 3rd Edition.*

MATH-ECT-304 (N6): Numerical Programming in Computational Software
Total Lectures: 50 Hrs. Marks: 50+10=60 Credit: 2.4

Introduction to Computational Software: MATLAB/MATHEMATICA/MAPLE basics: Basic computer programming, variables and constants, operators and simple calculations, formulas and functions, toolboxes, matrices and vectors, vectors and matrices, matrix operations and functions, exercises.
Software Programming: Algorithms and structures, scripts and functions (m/mb/mw-files), simple sequential algorithms, control structures, reading and writing data, file handling, personalized functions, toolbox structure, graphics, exercises.
Numerical Simulations: Simulation of systems of ordinary differential equations, *simulation of partial differential equations:* Wave equations, solution of nonlinear equations: Boundary value problems, Poisson and Laplace equations, solutions of two coupled (algebraic, trigonometric) nonlinear equations, roots of a polynomial etc., and exercises.

Recommended Books:

1. *B. R. Hunt, R. L. Lipsman, J. M. Rosenberg, A guide to Matlab for beginners and experienced users, Cambridge University Press, (1995).*
2. *A. Knight, Basics of Matlab and beyond, Chapman & Hall/CRC, (2000).*
3. *K. E. Lonngren, S. V. Savov, R. J. Jost, Fundamentals of Electromagnetics with MATLAB, SciTech Publishing, (2007).*

MATH-ECT-304(N7): Statistical Learning Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Supervised learning Overview of linear regression (LR), and statistical learning, supervised learning, variable types and simple approaches for predication, statistical models, classes of restricted estimates.
Linear methods for regression and classifications Linear methods for regression, LR models and least square, subset selection and coefficient shrinkage, linear methods for classifications, indicator matrix, separating hyper planes.
Regularizations and smoothing Basis expansions and regularizations, piecewise polynomial and splines, filtering and feature extraction, spline smoothening, wavelet smoothing, Kernel smoothers, local regression in R^p, Local likelihood and other models, radial basis functions and kernels.

Model assessment and selection Model assessment and bias, model complexity, optimism of training error rate, minimum description length, bootstrap and maximum likelihood methods, the EM algorithm.
Unsupervised learning Unsupervised learning, association rules, market basket example and analysis, cluster analysis, proximity matrices, algorithms, self-organizing maps, principal components, curves and surfaces, non-negative matrix factorization.

<u>Recommended Books:</u>

1. *J. H. Friedman, R. Tibshirani, T. Hastie, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Springer, 2nd Ed., (2009).*
2. *S. C. Gupta, V. K. Kapoor, Fundamentals of Applied statistics, Sultan Chand and sons, (2003).*
3. *T. Veerarajan, Probability Statistics and Random processes, TMH, First reprint, (2007).*

MATH-ECT-304(N8): Quantum Mechanics Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Origins of Quantum Theory: Inadequacies of classical mechanics; Planck's quantum hypothesis; Photoelectric effect; Compton experiment; Bohr model of hydrogenic atoms, Wilson-Sommerfeld quantization rule, Correspondence principle, Stern-Gerlach experiment (brief description and conclusion only).
Wave Aspect of Matter: de Broglie hypothesis; mater waves; uncertainty principle; double-slit experiment; Concept of wave function; Gedanken experiments.
Schrodinger Equation: Time-dependent Schrodinger equation; Statistical interpretation – conservation of probability, equation of continuity, expectation value, Ehrenfest theorem; Formal solution of Schrodinger equation – time-independent Schrodinger equation, stationary state, discrete and continuous spectra, parity.
Solutions of Schrodinger equation in one-dimension: Infinite potential box; Step potential; Potential barrier; Potential well.
Linear Harmonic Oscillator in One-dimension: Classical description; Schrodinger method of solution; Energy levels and wave functions; Planck's law.
Hydrogenic atoms: Schrodinger equation for hydrogenic atoms; Solution in spherical polar coordinates; Spherical Harmonics, Energy levels and wave functions; Radial probability density.
Mathematical Foundations of Quantum Mechanics: Concept of wave function space and state space; Observables; Postulates of quantum mechanics; Physical interpretations of the postulates – expectation values, Ehrenfest theorem, uncertainty principle.

<u>Recommended Books:</u>

1. *B. H. Bransden, C. J. Joachain, Quantum Mechanics, Prentics Hall, (2005).*
2. *D. J. Griffiths, Introduction to Quantum Mechanics, Pearson Prentics Hall, Upper Saddle River, NJ, (2005).*
3. *S. N. Ghoshal, Quantum Mechanics, S Chand & Company Ltd, Kolkata, (2002).*
4. *L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, (1968).*

MATH-ECT-403(M1): Signed Measure and Product Measure
Total Lectures: 50 Hrs. Marks: 50+10=60 Credit: 2.4

Signed Measure: definition and examples, signed measure spaces, signed measure as the integral of a semi-integrable measurable function, signed measure as the difference of two positive measures, partial monotonicity of signed measure, monotone convergence theorem for signed measure.
Decomposition of signed measure, positive, negative and null sets in a signed measure space, Hahn decomposition theorem, mutual singularity of two signed measures, Jordan decomposition of a signed measure, relation between Hahn and Jordan decompositions, Jordan decomposition theorem, total variation measure, theorems exhibiting connection between mutual singularity and total variation, absolute continuity of a signed measure relative to a position measure; basic results, Lebesgue decomposition theorem, uniqueness of Lebesgue decomposition, Radon-Nikodym theorem.
Product measure and product measurable spaces, estimation of product measure in terms of integrals, Fubini's theorem and Tonelli's theorem.

Recommended Books:

1. *J. Yeh, Lectures on Real Analysis, World Scientific.*
2. *H. L. Royden, Real analysis, Macmillan Publishing Co., Inc. 4th Edition, (1993).*
3. *S. K. Berberian. Measure and integration. Chelsea Publishing Company, NY, (1965).*
4. *G. de Barra, Measure Theory and integration, Wiley Eastern Ltd. (1981).*
5. *R. G. Bartle, The Elements of Integration, John Wiley & Sons, Inc. New York, (1966).*
6. *I. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi (1999).*

MATH-ECT-403(M2): Topological Algebra Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Topological rings and vector spaces: Definitions, examples, neighbourhood of zero, subring, ideals, quotients and projective limits of rings and vector spaces, completion of topological rings and vector spaces, Baire spaces, Summability, continuity of inversion, locally bounded vector spaces and rings, locally Retro bounded Division rings, Norms and Absolute values, Finite dimensional vector spaces, topological division rings, real valuations and valuation rings, discrete valuations, extensions of real valuations.

Recommended Books:

1. *D. Dikranjan, Introduction to topological groups.*
2. *S. Warner, Topological Rings, Elsevier Science Publishers.*
3. *I. F. Wilde, Topological Vector Spaces (Lecture notes).*

MATH-ECT-403(M3): Differential Topology Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

We will study topological properties of smooth manifolds. Among topics that we will cover are:
Manifolds and Smooth Maps: Definitions, Derivatives and Tangents, The inverse function theorem, Immersions, Submersions, Transversality, Homotopy and Stability, Sard's theorem and Morse functions, Embedding Manifolos in Euclidean space.

Transversality and Intersection: Manifolds with boundary, one-manifolds and some consequences, Transversality, Intersection theory mod 2, Winding numbers and the Jordan-Brouwer separation theorem, The Borsuk-Ulam theorem.
Oriented Intersection Theory: Motivation, Orientation, Oriented intersection Number, Lefschetz fixed-point theory, vector fields and the Poincaré-Hopf theorem, the Hopf degree theorem, The Euler characteristic and Trjangulations.

<u>Recommended Books:</u>

1. *J. W. Milnor, Topology from a differentiable viewpoint, The University Press of Virginia, (1965).*
2. *V. Guillemin, A. Pollack, Differential Topology, American Mathematical Society, (2010).*
3. *M. W. Hirsch, Differential Topology, Springer, (1997).*
4. *A. Mukherjee, Topics in Differential Topology, Hindustan Book Agency, (2005).*

MATH-ECT-403(M4): Analytic Number Theory Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Euler's summation formula, Average order of arithmetic functions like divisor function, Mobius function, Sigma function, Euler's phi function etc. Distribution of prime numbers, Discussion of Prime Number Theorem, Tauberian Theorems.
Dirichlet series, Multiplication of Dirichlet series, Euler products. Finite abelian groups and characters, Gauss sums associated with Dirichlet characters. Riemann zeta function and Dirichlet L-function, their zero free regions and functional equations. Dirichlet's Theorem for primes in an arithmetic progression.

<u>Recommended Books:</u>

1. *T. M. Apostol, Introduction to Analytic number theory, Springer-Verlag, (1976).*
2. *H. Davenport, Multiplicative Number Theory, Springer.*
3. *K. Chandrasekharan, Introduction to Analytic Number Theory, Springer-Verlag, (1968).*
4. *H. Iwaniec, E. Kowalski, Analytic Number Theory, American Mathematical Society Colloquium Publications 53, American Mathematical Society, (2004).*
5. *M. R. Murty, Problems in Analytic Number Theory, Springer.*
6. *E. C. Titchmarsh, 2nd edition revised by D. R. Heath-Brown, The Theory of Riemann Zeta function, Oxford Science Publications.*

MATH-ECT-403(M5): Advanced Complex Analysis-II Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Harmonic functions, Mean-value property, Poisson's integral formula, Dirichlet problem for a disc, Harnack's theorem.
Doubly periodic function, Weierstrass Elliptic function.
Meromorphic functions. Spaces of meromorphic functions, Marty's theorem, Zalcman's lemma, Montel's theorem, Runge's theorem, Existence of meromorphic functions with prescribed zeros and singular parts, Mittag-Leffler theorem.
Definitions of the functions N(r,a), m(r,a) and T(r,f). Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems. Order of growth, order of meromorphic function. Comparative growth of logM(r) and T(r).
Nevanlinna's second fundamental theorem. Estimation of S(r) (Statement only). Nevanlinna's theorem on defiant functions. Nevanlinna's five-point uniqueness theorem, Milloux theorem.

Recommended Books:

1. *W. K. Hayman, Meromorphic functions, Oxford University Press, 1964.*
2. *A. I. Markusevich, Theory of Functions of a Complex Variables, Vol. I & II, Printice-Hall, 1965.*
3. *L. V. Ahlfors, Complex Analysis, McGraw-Hill, 3rd Edn. 1979.*
4. *H. Cartan, Theory of Analytic Functions, Dover Publication, 1995.*
5. *M. Dutta, L. Debnath, Elements of the Theory of Elliptic and Associated Functions with Applications, World Press Pvt., 1965.*
6. *L. Yang, Value distribution theory, Springer-Verlag Berlin Heidelberg, 1993.*
7. *T. W. Gamelin, Complex Analysis, Springer, New York, 2001.*

MATH-ECT-403(M6): Advanced Functional Analysis-II Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Operators and C*-algebras. Two density theorems. Ideal, quotients, and representations.
C*-algebras of compact operators. Commutative C*-algebras and normal operators. Positive elements in C*-algebras. Quotients of a C*-algebra. Positive forms and representation: existence of representation of C*-algebras. The enveloping C*-algebra of an involutive Banach algebra. Ideals in C*-algebra. Spectrum of a C*-algebra.
Gelfand–Naimark Theorem: an arbitrary C*-algebra is isometrically *-isomorphic to a C*-algebra of bounded operators on a Hilbert space. von Neumann`s double commutant theorem and Kaplansky's density theorem. C-Algebras and Hilbert Space Operators. Ideals and Positive Functionals. Von Neumann Algebras. Representations of C-Algebras. Direct Limits and Tensor Products. K-Theory of C*-Algebras. Toeplitz operators.

Recommended Books:

1. *W. Arveson, An invitation to C*-Algebra, Springer-Verlag, New York, 1976.*
2. *G. J. Murphy, C*-Algebras and operator theory, Academic press, San-Diego, 2014.*
3. *K. R. Davidson, C*-Algebra by Example, American Mathematical Society, Providence Rhode Island, 2014.*

MATH-ECT-403(M7): Advanced Algebra Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Exact sequences, short and split exact sequences, tensor product of modules, definition and existence, projective modules, injective modules, direct sum of projective modules, direct product of injective modules.
Nil radical, Jacobson radical, Chinese remainder theorem, Rings and modules of fractions, local ring, characterizations of local ring, localization.
Noetherian and Artinian modules, composition series, Simple and Semisimple modules. Semisimple ring, characterizations of semisimple ring, Jacobson semisimple ring, regular ring, idempotents in regular rings, lifting of idempotents.
Prime and semiprime ideal, m-system and n-system, prime radical, prime rings, semiprime ring, ring derivation on prime and semiprime rings.

Recommended Books:

1. *D. S. Dummit, R.M. Foote, Abstract Algebra (3e), John Wiley and Sons (Asian reprint).*
2. *B. Singh, Basic Commutative Algebra, World Scientific, 2011.*
3. *W. A. Adkins, S. H. Weintraub, Algebra, Springer-Verlag, 1999.*
4. *M. F. Atiyah, I.G. MacDonald, Introduction to commutative Algebra, CRC Press, 2019.*
5. *D. Eisenbud, Commutative Algebra, Springer, USA, 2004.*
6. *C. Musili, Representations of finite groups, Hindustan Book Agency, India, 2011.*

7. *B. Steinberg, Representation theory of finite groups, Springer (India reprint 2015).*
8. *J. J. Rotman, Advanced Modern Algebra, Springer (Indian reprint 2016).*

MATH-ECT-403(M8): Modular Forms Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

The full modular group $SL_2(\mathbb{Z})$ and its congruence subgroups, the upper half-plane H, Action of groups on H, Fundamental domains.
Modular forms for $SL_2(\mathbb{Z})$, modular forms for congruence subgroups, Eisenstein series, cusp forms. Differential operators. Structure of the ring of modular forms.
Hecke operators and Euler product for modular forms. The L-function of a modular form, functional equations. Theta functions, transformation formula, sums of four squares.

Recommended Books:

1. *S. Lang, Introduction to Modular Forms, Springer-Verlag, 1995.*
2. *J. P. Serre, A Course in Arithmetic, Graduate Texts in Mathematics 7, Springer-Verlag, 1973.*
3. *N. Koblitz, Introduction to Elliptic Curves and Modular Forms, Graduate Texts in Mathematics 97, Springer-Verlag, 1993.*
4. *J. H. Bruinier, G. van der Geer, G. Harder, D. Zagier, The 1-2-3 of Modular Forms, Universitext, Springer-Verlag, 2008.*
5. *F. Diamond, J. Shurman, A First Course in Modular Forms, Graduate Texts in Mathematics 228, Springer-Verlag, 2005.*
6. *M. Ram Murty, Problems in the theory of modular forms, Hindustan book agency, 2015.*

MATH-ECT-403(M9): Algebraic Geometry Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Preliminaries: Some category theory, Motivation, Categories and functors, Universal properties determine an object up to unique isomorphism, Limits and colimits, Adjoints, An introduction to abelian categories, Spectral sequences, Sheaves. Motivating example: The sheafof differentiable functions. Definition of sheaf and presheaf, Morphisms of presheaves and sheaves, Properties determined at the level of stalks, and sheafification, Sheaves of abelian groups, and OX-modules, form abelian categories, The inverse image sheaf, Recovering sheaves from a "sheaf on a base".
Schemes: Toward affine schemes: the underlying set, and topological space, Toward schemes, The underlying set of affine schemes, Visualizing schemes I: generic points, The underlying topological space of an affine scheme, A base of the Zariski topology on Spec A: Distinguished open sets, Topological (and Noetherian) properties, The function I(·), taking subsets of Spec A to ideals of A, The structure sheaf, and the definition of schemes in general, The structure sheaf of an affine scheme, Visualizing schemes II: nilpotents, Definition of schemes , Three examples, Projective schemes, and the Proj construction, Some properties of schemes, Topological properties, Reducedness and integrality, Properties of schemes that can be checked "affine-locally", Normality and factoriality, Where functions are supported: Associated points of schemes.
Morphisms: Morphisms of schemes, Introduction, Morphisms of ringed spaces, From locally ringed spaces to morphisms of schemes, Maps of graded rings and maps of projective schemes, Rational maps from reduced schemes, Representable functors and group schemes, The Grassmannian (initial construction), Useful classes of morphisms of schemes, An example of a reasonable class of morphisms: Open embeddings, Algebraic interlude: Lying

Over and Nakayama, A gazillion finiteness conditions on morphisms, Images of morphisms: Chevalley's theorem and elimination theory, Closed embeddings and related notions, Closed embeddings and closed subschemes, More projective geometry, Smallest closed subschemes such that ... ,Effective Cartier divisors, regular sequences and regular embeddings, Fibered products of schemes, and base change, They exist, Computing fibered products in practice, Interpretations: Pulling back families, and fibers of morphisms, Properties preserved by base change, Properties not preserved by base change, and how to fix them, Products of projective schemes: The Segre embedding, Normalization, Separated and proper morphisms, and (finally!) varieties, Separated morphisms (and quasi separatedness done properly), Rational maps to separated schemes, Proper morphisms.
Geometric Properties: Dimension and Smoothness: Dimension, Dimension and codimension , Dimension, transcendence degree, and Noether normalization, Codimension one miracles: Krull's and Hartogs's Theorems, Dimensions of fibers of morphisms of varieties, Proof of Krull's Principal Ideal and Height Theorems, Regularity and smoothness, The Zariskitangent space, Regularity, and smoothness over a field, Examples, Bertini's Theorem, Discrete valuation rings: Dimension, Noetherian regular local rings, Smooth (and etale) morphisms (first definition), Valuative criteria for separatedness and properness, More sophisticated facts about regular, local rings, Filtered rings and modules, and the Artin-Rees Lemma.

Recommended Books:

1. *M. F. Atiyah, I. G. MacDonald, Introduction to Commutative Algebra, Addison-Wesley, (1969).*
2. *J. S. Milne, Commutative Algebra, Cambridge University Press, (2017).*
3. *J. S, Milne, Fields and Galois Theory, Taiaroa Publishing, (2017).*
4. *R. Hartshorne, Algebraic Geometry, Springer, (1977)*
5. *I. R. Shafarevich, Basic Algebraic Geometry, Springer, (1994).*

MATH-ECT-403(M10): Category Theory Marks: 50+10=60 Credit: 2.4

Total Lectures: 50 Hrs.

Category, Diagrams, Monic, Epic, Initial & terminal objects, Sections & retractions, large, small & locally small category, Subcategory, Opposite category, Product category.
The duality principle, Product & coproduct of objects, Pullback & pushout, Equalizers & coequalizers, Limit & colimit.
Functors (covariant & contravariant), Full, faithful, equivalence & isomorphism functors, Forgetful functors, Hom functors, Representable functors, Product of functors.
Natural transformations, vertical & horizontal compositions, Functor categories, Category of categories, Exponentials of categories, Yoneda's lemma and its applications, Universal and adjoint functors, Equivalence of categories.

Recommended Books:

1. *S. Awodey, Category Theory, Oxford University Press, (2010).*
2. *S. M. Lane, Categories for the working Mathematician, Springer, (2013).*
3. *H. Simmons, An Introduction to Category Theory, Cambridge University Press, (2011).*
4. *T. Leinster, Basic Category Theory, Cambridge University Press, (2014).*

MATH-ECT-403(M11): General Theory of Integration Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Tagged Gauge Partitions. Definitions, Cousins Theorem, Right-left Procedure, Straddle Lemma, Application in continuity, Intrinsic Power. Henstock–Kurzweil Integral. Definition and basic properties. Fundamental Theorem, Saks-Henstock Lemma, Inclusion of the Lebesgue integral. Squeez Theorem, Vitali- Covering Theorem, Differentiation Theorem, Characterization Theorem.

Recommended Books:

1. *R. G. Bartle, A Modern Theory of Integration, AMS*
2. *D. S. Kurtz, C. W. Swartz, Theories of Integration, World Scientific.*
3. *L. P. Yee, Lanzhou Lectures on Henstock Integration, World Scientific.*
4. *A.G. Das, The Riemann, Lebesgue and General Riemann Integrals, Narosa.*
5. *R. Henstock, The general Theory of integration, Clarendon Press.*

MATH-ECT-404(N1): Boundary Integral Equations Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Prerequisite: Partial Differential Equations
Green representation formula, Green's function of Laplace equation in 1D, 2D, and 3D, Green's function of Helmholtz equation, integral representation, Hypersingular integrals, generalized single and double layer representations, boundary potentials, Laplace and Poisson problems, boundary integral equations of the Dirichlet and Neumann problems (interior and exterior), mixed and Robin Boundary Value Problems, interface problem, Helmholtz equation with Dirichlet and Neumann Boundary Value Problem, low-frequency behaviour.
Biharmonic Equation: Calderon’s projector, Boundary Value Problems, Boundary Integral Equations, Lame` equations: Dirichlet, Neumann, and mixed Boundary Value Problems, derivation of the Boundary Element Method in 2D and 3D, the Boundary Integral Method for Stokes equation, inhomogeneous, nonlinear, and time dependent problems.

Recommended Books:

1. *C. Pozrikidis, Boundary Integral and Singularity Methods for Linearized Viscous Flow, Cambridge university press, 1992.*
2. *C. Pozrikidis, A Practical Guide to Boundary Element Methods with the Software Library BEMLIB, CRC Press, 2002.*
3. *S. Rjasanow, and O. Steinbach, The Fast Solution of Boundary Integral Equations, Springer Science & Business Media, 2007.*
4. *G. C. Hsiao, and W. L. Wendland, Boundary Integral Equations, Springer, 2008.*
5. *V. Maz'ya, and A. Soloviev, Analysis IV-Linear and Boundary Integral Equations, Springer-Verlag, 1991, pp. 127-222.*

MATH-ECT-404(N2): Mathematical Ecology Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Mathematical Models of Population Biology or Ecology: ***Mathematical models****:* Deterministic and stochastic, single species population models, P-V logistic equation, population growth model- an age structured model.

Interactions between two species: Host-Parasite type of interactions, competitive type of interactions.
Trajectories of interactions of H-P and competitive types between two species, effect of migration on H-P interactions, some consequences of Lotka-Volterra equations, generalized L-V equations, constant of motion in the dynamical system, stochastic processes and need of stochastic models, pure birth process, pure death process, birth and death process, linear birth-death-immigration-emigration processes, effects of both immigration and emigration on the dynamics of population.
Biological mechanisms responsible for time-delay, discrete and continuous time-delay, the single species logistic model with the effect of time-delay, stability of equilibrium position for the logistic model with general delay function, stability of logistic model for discrete time lag, time-delayed H-P model together with their stability analysis.
Mathematical Theory of Epidemics: Introduction, some basic definitions, simple epidemic model, general epidemic model, Kermack-McKendrik threshold theorem, recurring epidemic model, a comparative study of these models.
Control of an epidemic, stochastic epidemic model without removal, models having multiple infections.
Epidemic model with multiple infections, stochastic epidemic model with removal, stochastic epidemic model with removal, immigration and emigration, special discussion on the stochastic epidemic model with carriers.
Simple extensions of SIR model: Different case studies (i) Loss of immunity, (ii) Inclusion of immigration and emigration, (iii) Immunization, SIR endemic disease model.

Recommended Books:

1. *J. D. Murray, Mathematical Biology, Springer-Verlag, Berlin, (1989).*
2. *X. Q. Zhao, Dynamical Systems in Population Biology, Canadian Mathematical Society,(2003)*
3. *J. N. Kapur, Mathematical Models in Biology and Medicine, East West Press Pvt Ltd, (1985)*
4. *R. Habermann, Mathematical Models, Prentice Hall, (1977).*
5. *E. C. Pielou, An Introduction to Mathematical Ecology, Wiley, New York, (1977).*
6. *R. Rosen, Foundation of Mathematical Biology (vol. I& II), Academic Press, (1972)*
7. *M. Kot, Elements of Mathematical Ecology, Cambridge University Press, (2003).*
8. *R. M. Andersson, R. M. May, Infectious Diseases of Humans, Oxford University Press, (1992)*

MATH-ECT-404(N3): Biofluid Mechanics Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Arterial Biomechanics: Importance of studies on the mechanics of blood vessels, structure and functions of blood vessels, mechanical properties, viscoelasticity, linear discrete viscoelastic (spring-dashpot) models: Maxwell fluid, Kelvin solid, Kelvin chains and Maxwell models, creep compliance, relaxation modulus, hereditary integrals, Stieltjes Integrals.
Constituents of blood, structure and functions of the constituents of blood, mechanical properties of blood, equations of motion applicable to blood flow, non-Newtonian fluids - Power law, Bingham Plastic, Herschel-Bulkley and Casson fluids, steady non- Newtonian fluid flow in a rigid circular tube, Fahraeus-Lindqvist effect, pulsatile flow in both rigid and elastic tubes, blood flow through arteries with mild stenosis, shear stress on surface of the stenosis, two-layered flow in a tube with mild stenosis.

Large deformation theory, various forms of strain energy functions, the base vectors and metric tensors.
Green's deformation and Lagrangian strain tensors, cylindrical model, constitutive equations for blood vessels, equations of motion for the vascular wall.
Biological Diffusion and Diffusion-Reaction Models: Fick's laws of diffusion, one-dimensional diffusion model and its solution, some solutions of two dimensional diffusion equation, various modifications of diffusion equation to diffusion-reaction models arising in pharmacokinetics and ecology.
Hemodialyzer and dialysis of blood: Basic equations for a circular-duct and a parallel-plate dialyser, Peclet number, Sherwood number, solutions of basic equation for a circular-duct dialyser by (i) separation of variables method and (ii) Galerkin's method, solution for parallel-plate dialyser.

Recommended Books:

1. *J. N. Kapur, Mathematical Models in Biology and Medicine, East West Press Pvt Ltd , (1985).*
2. *Y. C. Fung, Biomechanics of Soft Biological Tissues, Springer, New York, (1993).*
3. *R. Habermann, Mathematical Models, Prentice Hall (1977).*
4. *E. C. Pielou, An Introduction to Mathematical Ecology, Wiley, New York, (1977).*
5. *R. Rosen, Foundation of Mathematical Biology (vol. I& II), Academic Press, (1972).*
6. *W. Flugge, Viscoelasticity, Springer-Verlag, (1975).*
7. *M. Zamir, E L Ritman, The Physics of Pulsatile Flow, Springer, (2000).*
8. *D. A. MacDonald, Blood Flow in Arteries, The Williams and Wilkins Company, Baltimore (1974).*
9. *J.N. Mazumdar, Biofluid Mechanics, World Scientific (2015).*

MATH-ECT-404(N4): General Theory of Relativity and Cosmology
Total Lectures: 50 Hrs. **Marks: 50+10=60 Credit: 2.4**

General Theory of Relativity: Geometry of curved space-time, equivalence principle.
Space-time symmetries: killing vectors & their properties. Homogeneity & isotropy, Curvature of maximally symmetric spaces.
Energy-momentum tensor, energy conditions, perfect fluid.
Einstein's field equations: approach of Heuristic derivation and from Action Principle, Schwarzschild solution, Birkhoff's theorem, singularities, Weak field approximation of gravity: Newtonian limit, Geodesics in Schwarzschild space-time.
Experimental test of General Relativity: Deflection of light, precession of planetary orbits, Radar echo delay, Gravitational lenses.
Black Holes: Schwarzschild black hole, mass, charge and spin; Event horizon, Apparent horizon. Kerr Black hole (rotating).
Cosmology: Homogeneous and isotropic space-time, FLRW universe, Cosmological principle, Weyl's postulate,
Cosmic dynamics: Friedmann equation, acceleration equation, Hubble's law in relativistic cosmology, Gravitational red-shift and Cosmological red-shift.
Cosmological model of universe: Evolution of energy density of the matter content, model with vanishing and non-vanishing cosmological constant, The Friedmann dust universe, Open and Closed model of universe, Empty universe, The Einstein static model, the de Sitter model, Evolution of universe with multiple matter content.
Cosmological parameters: Hubble parameter, Luminosity distance, Angular-diameter distance, deceleration parameter, equation of state parameter.

Cosmological observations: The Cosmic Microwave Background (CMB) and recent observations.
Inflation and very early universe: The flatness problem, The Horizon problem, The Monopole problem, The inflation solution.
Dark Matter: Observational evidences, Visible matter, Dark matter in galaxies, Dark Matter in clusters, Gravitational lensing.
Dark Energy: Failure of Standard Cosmological model: Theoretical introduction to Dark Energy, Cosmological Constant Λ as Dark Energy, ΛCDM, Cosmological constant problem and coincidence problem, Dynamical Dark Energy models based on scalar field: Quintessence, k-essence, tachyon, phantom.

<u>Recommended Books:</u>

1. *L. Ryder, Introduction to General Relativity, Cambridge University Press, 2009.*
2. *B. F. Schutz, A First Course in GENERAL RELATIVITY, Cambridge University Press, 2009.*
3. *E. Poisson, A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics, Cambridge University Press, 2004.*
4. *S. M. Carroll, An introduction to general relativity: Space-time and Geometry, Pearson, Addison Wesley, 2004.*
5. *R. d' Inverno, Introducing Einstein's Relativity, Oxford University Press 2012.*
6. *S. Chakraborty, A Treatise on Differential Geometry and its role in Relativity Theory-Book. ref: arXiv:1908.10681[gr-qc] (2019).*
7. *A. K. Raychaudhury, S. Banerji and A. Banerjee, General Relativity, Astrophysics and Cosmology, Springer Science & Business Media, 2003.*
8. *B. Ryden, Introduction to Cosmology, Cambridge University Press, 2017.*
9. *S. Dodelson, Modern Cosmology, Elsevier Science Publishing Co Inc. 2020.*
10. *J. V. Narlikar, An Introduction to cosmology, Cambridge University Press, 2002.*

MATH-ECT-404(N5): Lie Theory of Ordinary and Partial Differential Equations

Total Lectures: 50 Hrs. Marks: 50+10=60 Credit: 2.4

Lie Group of Transformations and Infinitesimal Transformations: Introduction, Lie group of transformations, infinitesimal transformations, point transformations and extended transformations (prolongations), multi-parameter Lie group of transformations and Lie algebras, mappings of curves and surfaces, local transformations.
Ordinary Differential Equations: Elementary examples, first order ODEs, invariance of second and higher order ODEs under point symmetries, reduction of order of ODEs under multi-parameter Lie group of point transformations.
Invariance of a PDE: Introduction, determining equations for symmetries of a k^{th}-order PDE, invariance of scalar PDE, elementary examples.
Invariant Solution of PDEs: Invariant solutions, example, invariance for a system of PDEs, determining equations for symmetries of a system of PDEs, examples,
Application to Boundary Value Problems: Formulation of invariance of a Boundary Value Problem for a scalar PDE, incomplete invariance for a linear scalar PDE, incomplete invariance for a linear system of PDEs.

<u>Recommended Books:</u>

1. *G. W. Bluman, S. C. Anco, Symmetry and Integration Methods for Differential Equations, Springer, (2002).*
2. *P. J. Olver, Application of Lie Groups to Differential Equations, Springer, (2000).*

3. *N. H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations John Wiley & Sons, (1999).*
4. *H. Stephani, Differential Equations: Their Solution Using Symmetries, Camb. Univ. Press, (1989).*
5. *G. Baumann, Symmetry Analysis of Differential Equations with Mathematica, Springer (Telos), (2000).*

MATH-ECT-404(N6): Nonlinear Optimization Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Convex set, Convex function, Generalized convex functions. Fritz John and Karush- Kuhn – Tucker optimality condition, duality, Convex programming problems, Quadratic programming, Fractional programming, Separable programming, Non-linear integer programming. Constrained Optimization: One dimensional search methods, Multi-dimensional search methods. Unconstrained optimization: Conjugate gradient method, Generalized reduced gradient methods, Method of feasible direction.

Recommended Books:

1. *N. Deo, Graph Theory with Applications to Engineering and Computer Science, PHI.*
2. *S. S. Rao, Engineering Optimization: Theory and Practice, New Age International Pvt. Ltd., 3rd Edition, (1998).*

MATH-ECT-404(N7): Computational Statistics Marks: 50+10=60 Credit: 2.4
Total Lectures: 50 Hrs.

Analysis of Variance, one-way and two-way classification, Concept of design of experiment. Some standard designs: completely randomized design, randomized block design, Latin Squares, Graeco Latin Squares, and factorial designs, confounding and blocking in factorial designs, fractional factorial designs. Simple and multiple regression models. Classical techniques of time series analysis, smoothing and decomposition. Analysis of covariance model.

Recommended Books:

1. *G. H. Givens, Jennifer A. Hoeting, Computational Statistics, John Wiley & Sons, (2012).*
2. *J. E. Gentle, W. K. Härdle, Y. Mori, Handbook of Computational Statistics: Concepts and Methods, Springer Science & Business Media, (2012).*
3. *J. E. Gentle, Computational Statistics, Springer Science & Business Media, (2009).*
4. *W. L. Martinez, A. R. Martinez, Computational Statistics Handbook with MATLAB, CRC Press, (2015).*

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