UNIVERSITY OF NORTH BENGAL

समानो मन्त्रः समितिः समानी

Syllabus for

M.Phil.

in

MATHEMATICS

Department of Mathematics, University of North Bengal, Raja Rammohunpur, P.O.- N.B.U., Dist- Darjeeling, West Bengal, India, Pin-734013

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18/12/2020.

Signature of HOD
Department of Mathematics, NBU

HEAD
Department of Mathematics
University of North Bengal

Structure of Syllabus for M.Phil. in Mathematics

Semester-I

Course Code	Course Type	Course Name	Full Marks (External)	Full Marks (Practical)	Full Marks (Internal)	Full Marks (Total)	Credit
MPHILMA-101	**Theory**	**Research Methodology: Research Foundation**	**20**		**5**	**25**	**2**
MPHILMA-102	**Practical**	**Research Methodology: Computer Application in Research**		**20**	**5**	**25**	**2**
MPHILMA-103	**Theory**	**Elective:** Any two papers to be chosen from **Table-I**, based on research interest of the students and availability of suitable teachers/ Supervisors.	**40** **+** **40**		**10** **+** **10**	**50** **+** **50**	**4** **+** **4**
		Total	**120**		**30**	**150**	**12**

Semester-II

Course Code	Course Type	Course Name	Full Marks (External)	Full Marks (Practical)	Full Marks (Internal)	Full Marks (Total)	Credit
MPHILMA-201	**Theory**	**Elective:** Any three papers to be chosen from **Table-II**, based on research interest of the students and availability of suitable teachers/Supervisors.	**40** **+** **40** **+** **40**		**10** **+** **10** **+** **10**	**50** **+** **50** **+** **50**	**4** **+** **4** **+** **4**
		Total	**120**		**30**	**150**	**12**

Semester-III

Course Code	Course Type	Course Name	Full Marks (External)	Full Marks (Practical)	Full Marks (Internal)	Full Marks (Total)	Credit
MPHILMA-301	Theory	Preliminary Dissertation			75	75	6
MPHILMA-302	Theory	Viva-voce			25	25	2
		Total			100	100	8

Semester-IV

Course Code	Course Type	Course Name	Full Marks (External)	Full Marks (Practical)	Full Marks (Internal)	Full Marks (Total)	Credit
MPHILMA-401	Theory	Final Dissertation			75	75	6
MPHILMA-402	Theory	Viva-voce			25	25	2
		Total			100	100	8

Table: I

Elective Papers for MPHILMA-103 (M1) to MPHILMA-103 (M18)	
Elective Paper Sub-Code	**Title of the Paper**
M1	**Theory of Convergence and Topological Hyperalgebra**
M2	**Basic Theory of Entire and Meromorphic Functions**
M3	**Measure Theory**
M4	**Summability Methods-I**
M5	**Topics in Analytic Number Theory**
M6	**Topological Indices of Graphs**
M7	**Advanced Modern Algebra-I**
M8	**Finite and Boundary Element Methods**
M9	**Similarity Transformations and Perturbation Theory**
M10	**Advanced Numerical Techniques and MATLAB**
M11	**Modern Theory of Partial Differential Equations**
M12	**Advanced Trends in Continuum Mechanics**
M13	**Advanced Fluid Mechanics**
M14	**Mathematical Methods**
M15	**Optimization Techniques**
M16	**Decision Theory**
M17	**Differential Geometry of Manifolds**
M18	**Mathematical Theory of Elasticity**

Table: II

Elective Papers for MPHILMA-201 (N1) to MPHILMA-201 (N19)	
Elective Paper Sub-Code	**Title of the Paper**
N1	**General Theory of Integration**
N2	**Value Distribution Theory of Meromorphic Functions**
N3	**Summability Methods-II**
N4	**Advanced Modern Algebra-II**
N5	**Modular Forms**
N6	**Complex Manifolds**
N7	**Contact Manifolds**
N8	**General Theory of Relativity and Cosmology**
N9	**Advanced Quantum Mechanics**
N10	**Boundary Layer Theory and Turbulence**
N11	**Chaos Theory and Fractals**
N12	**Non-linear Partial Differential Equations**
N13	**Fuzzy Sets and Applications**
N14	**Mathematical Logic**
N15	**Multivariate Analysis**
N16	**Numerical Optimization**
N17	**Lie Groups and Lie Algebras**
N18	**Queuing Theory**
N19	**Data Structure and Algorithm**

Detailed Syllabus

Semester-I

MPHILMA-101: Research Methodology: Research Foundation **Full Marks: 25 Credit: 2**

Introduction- Meaning, purpose, objectives, characteristics, motivation, significance, types of research; approaches, process, methods and methodology used in research ; criteria of good research; research methods in general and Mathematical Sciences in particular.

The Research Problem – Research problems and sub-problems identification, stating, defining, techniques involved in defining problem.

Interpretation and Report Writing- Techniques of interpretation- significance, types, steps, checklist/precautions and characteristics of research documentation i.e. reviews, treatise, monographs, abstracts, articles, technical reports, white papers, research papers, thesis etc.- issues and techniques of writing project proposals, paper presentation and soft skills.

Ethical issues and Professional Conduct- Ethics in general, Professional Ethics, ethical Issues and their significance those arise from Computer Technology, General Moral Imperatives, Concepts and issues related to plagiarism and Intellectual Property Rights

Literature Review – Importance of Literature review in defining a problem, including literature in research proposal, critique, survey & peer review process, identifying gap areas from literature review; Major Research areas, Journals, Publication, Conferences and Status of Research in the field of Mathematical Sciences.

Extensive review work has to be undertaken by the candidate in relevance to his/her topic of research interest. The candidate will be required to submit a written report on the survey to be evaluated by the Departmental Council. Candidate will also be required to deliver a seminar lecture on the survey work.

References:

1. *Research Methodology Methods Techniques by C. R. Kothari, Wishwa Prakashan Publishers.*
2. *An introduction to Research Methodology by Garg, B.L., Karadia, R., Agarwal, f. and Agarwal, U.K., RBSA Publishers.*
3. *Research Methodology Sinha, S.C. and Dhiman, A.K.,Ess Publications. 2 volumes.*
4. *Research Methods: the concise knowledge base by Trochim, W.M.K., Atomic Dog Publishing.*
5. *How to Write and Publish a Scientific Paper by Dey, R.A., Cambridge University Press.*
6. *Conducting Research Literature Reviews: From the Internet to Paper by Fink, A., Sage Publications*
7. *Proposal Writing by Coley, S.M. and Scheinberg, C.A., Sage Publications, 1990.*
8. *Handbook on Intellectual Property Law and Practice by Subbarau NR, S Viswanathan, Printers and Publishing Private Limited*
9. *Research Methodology by Dalip Kumar Bhattacharya*
10. *Research Methodology by C.H Chaudhary, RBSA Publication*
11. *Statistical Techniques by S.P. Gupta, Sultan Chand & Sons*
12. *An Introduction to Multivariate Statistical Analysis by Andreson T. W., Wiley Eastern Pvt., Ltd., New Delhi.*
13. *WWW(Web Sources)*

MPHILMA-102: Research Methodology: Computer Application in Research

Full Marks: 25 Credit: 2

Historical background of LATEX and its pronunciation:
Creating a new document, Opening and saving a Doument, Use of document classes like article.cls, amsart.cls, books.cls, report.cls; Adding pckages like amssymb, amsmath, amsthm, amsfonts, graphics, graphix, times;

Entering Text, Editing Text

Entering and Editing Mathematics: Entering Mathematical Characters, Entering Mathematical Objects, Entering Mathematics with Fragments, Using Body Math, Editing Mathematics

Formatting Your Document: Formatting with tags: theorem, definition, corollary, lemma, proposition, example, acknowledgement, axiom, proposition and changing their styles; Formatting the Page, Changing body text point size, paper size, orientation, title page, two column category, equation number position; Use of unit of measurement: ex, em, pt, cm, mm, in; Definition of commands : \newcommand, \renewcommand, \setcounter, \addcounter, \newenvironment, \renewenvironment; Introduction to spacing: horizontal spacing, vertical spacing, small space, big space etc; Different pagebreaks: newline, line break; Making section, subsection etc, Alignment of texts, pictures, creating post script files (*.ps, *.eps) and inserting them into Tex files.

Saving Your Documents: **Saving Portable LATEX Files, Exporting Files.**

Previewing and Printing Your Document: Creating PDF Files, Exporting Documents as HTML Files.

Special features of Scientific Workplace: Computing and Plotting, Evaluate and Evaluate Numerically, Factor, Combine, Expand, Simplify, Equality, Solve Exact and Numeric, Work with Polynomials, 2-D and 3-D Plots, Compute in Place, Perform Matrix Operations, Solve Differential Equations, Compute Statistics, Compute with Units of Measure, Create Exams and Quizzes.

Typesetting Your Document: Understanding the Typesetting Process, Typeset Previewing and Typeset Printing, Understanding the Appearance of Typeset Documents, Creating Typeset Document Elements, Creating Cross-References, Creating Notes, Creating Bibliographies and Citations, Obtaining More Information about Typesetting.

Basics of operating systems: handling different operating systems, Literature survey using web, handling search engines.

Computer usage for collecting/analyzing data: simulations using fortran/ C/ Mathematica/ MATLAB/Mathcad----Solving Differential equation, numerical solution of differential equation, Integration, Solution of Algebraic equation, numerical solution of differential equation, Integration, Solution of Algebraic equation; Plotting of solution curve of system of differential equations, Two- and three-dimensional plotting, plotting data, using options, and creating dynamic and interactive graphics.

Preparing presentations:

i) Research papers: Using word processing software – MS Word/Latex/others, Drawing graphs and diagrams – Origin/Xmgrace/Excel/others.

ii) Seminar presentations – Beamer/Power point for oral and poster presentations.

Use of Acrobat reader, Texnic Centre, WinEdt, Ghost View; Conversion of graphics in different formats by Ghost View.

<u>References:</u>

1. *A document preparation system- Leslie Lamport, Addison Weseley Publisher Company, 1994*
2. *Getting Started with Scientific Workplace®, Scientific Word® & Scientific Notebook® Version 5- Susan Bagby, Publisher: MacKichan Software*
3. *The Computer Science of TeX and LaTeX by Victor Eijkhout*

4. *LATEX for Biginners: Workbook [Available at http://www.docs.is.ed.ac.uk/skills/documents/3722/3722-2014.pdf]*
5. *LATEX Tutorials A PRIMER Indian TEX Users Group Trivandrum, India 2003 September Edited by E. Krishnan [Available at https://www.tug.org/twg/mactex/tutorials/ltxprimer-1.0.pd]*
6. *LaTeX Wikibooks.org/wiki/LaTeX]*
7. *LaTeX Beginner's Guide by Stefan Kottwitz*
8. *Getting Started with LaTeX by David R. Wilkins*

M1: Theory of Convergence and Topological Hyperalgebra **Full Marks: 50 Credit: 4**

Theory of Convergence

A Generalized Statistical Convergence via Ideals, I and I*convergence in topological spaces, A note on I-convergence and I*-convergence of sequences and nets in topological space, I and I* convergence of nets, On I-convergence of nets in locally solid Riesz space, Some further results on I-Cauchy sequences and condition (AP), I-Lambda statistical convergence in topological groups, When I-Cauchy nets in complete uniform spaces are I-convergent, Some Further results on Ideal Convergences in Topological Spaces, I-Lambda Statistically convergent sequences in topological groups, On I-Cauchy nets and Completeness, When I-Cauchy nets in complete uniform spaces are I-convergent, I^K- convergence, I^K-Cauchy Functions, Extending Asymmetric convergence and Cauchy conditions using ideals.

Topological Hyperalgebra:

Introduction to Topological Groups and Semigroups:

Semitopological groups: The concept of a semitopological group, neighborhood system of identity, constructions of new semitopological groups from old, locally compact semitopological groups.
Topological group: Translation in topological groups, neighborhood system of identity, separation axioms in topological groups, uniform structure on a topological group, sub groups, quotient groups, locally compact groups, continuous and open homomorphism, open homomorphism and closed graph theorems.

Introduction to Topological Rings:

Definition, examples of topological rings, topological modules, vector space, and algebras, neighborhoods of zero, subrings, ideals, submodules.

Metrizable groups, completions of commutative Hausddorff groups, completions of topological ring and modules, locally bounded modules and rings.

Introduction to algebraic hyperstructures:

What algebraic hyperstructures are? A historical development of algebraic hyperstructures.
The hypergroup of Marty: Definitions, examples, subhypergroups, Some kinds of subhypergroups, homomorphism of hypergroups, Join spaces, Canonical hypergroups, Polygroups, regular and strongly regular relations, the fundamental relation, complete parts, the heart of a hypergroup, complete semihypergroups, complete hypergroups, Polygroups: Difinition, examples, extension of polygroups by polygroups, subpolygroups and quotient polygroups , isomorphism theorems of polygroups, generalized permutation, permutation polygroups, representation of polygroups, solvable polygroups, nilpotent polygroups.Thehyperring of Krasner: Definition and constructions of Krasner hyperrings, hyperideals, quotient hyperrings and homomorphisms, special hypeideals.

Introduction to topological algebraic hyperstructures:

Topological Hypergroups in the sense of Marty: Definitions, examples, topological subhypergroup, compact Hausdorff topological hypergroup.

Topological Polygroups; Definition, examples, results, subpolygroups of a topological polygroup, isomorphism theorems, rolls of complete parts in topological polygroups, left big subsets of topological polygroups.

Topological Polygroups: Definition, examples, related identity, some results.

References:

1. *T. Husain, Introduction to Topological Groups, W. B. Saunders Company, 1966.*
2. *A. Arhangel'skii, M. Tkachenko, Topological Groups and Related Structures, Atlantis Press/Word Scientific, 2008.*
3. *G. McCarty, Topology: An Introduction with Application to Topological Groups, Dover, 2011.*
4. *B. Davvaz, V. Leoreanu-Fotea, Hyperring Theory and Applications, International Academic Press, 115, Palm Harber, USA, 2007.*
5. *B. Davvaz, Polygroup theory and Related Systems, Word Scientic Publishing Co. Pte. Ltd., Hackensack, NJ, 2013.*
6. *D. heidari, B. Davvaz, S. M. S. Modarres, Topological hypergroups in the sense of Marty, Communications in Algebra, 42 (2014) 4712-4721.*
7. *D. Heidari, B. Davvaz, S. M. S. Modarres, Topological polygroups, Bull. Malays. Math. Sci. Soc., 39 (2016) 707-721.*
8. *M.S Shadkami, M.R. AhmadiZand, B. Davvaz, The Role of Complete Parts in Topological Pologroups, Int. J. Anal. Appl., 11(1)(2016) 54-60.*
9. *M. SalehiShadkami, M. R. AhmadiZand and B. Davvaz, Left big subsets of topological polygroups, Filomat, 30:12 (2016), 3139-3147.*
10. *M. Singha, K. Das, B. Davvaz, On Topological Complete Hypergroups, Filomat 31 (2017), no. 16, 5045–5056.*

M2: Basic Theory of Entire and Meromorphic Functions **Full Marks: 50 Credit: 4**

Elementary theory of entire and meromorphic functions, Poission –Jensen formula, Nevanlinna characteristic function, Nevanlinna's first fundamental theorem, Cartan's identity, Orders of growth, Comparative growth of T(r) and log M(r), Weierstrass product, Representation of a meromorphic function in terms of its zeros and poles, Nevanlinna's second fundamental theorem, Various types of deficiencies of meromorphic functions and their properties.

References:

1. *W. K. Hayman: Meromorphic functions, The claredon Press, Oxford, 1964.*
2. *C. C. Yang and H. X. Yi: Uniqueness Theory of Meromorphic Functions, Science Press, Beijing, 2003.*
3. *G. Valiron: Lectures on the general theory of integral functions, Chelsea Publishing Company, 1949.*

M3: Measure Theory **Full Marks: 50 Credit: 4**

Lebesgue Measure space on the Euclidean space.Lebesgue Outer Measure on the Euclidean space. Definition and properties. Regularity properties; Borel measure space of n – dimension; Completion of Borel Measure space; Estimation of Lebesgue measure on $\mathbb{R}^n$byclosed and compact sets and approximation by open sets.

Hausdorff Measures, Regularity and Hausdorff dimension, Hausdorff measures of integral and functional dimensions.

References:

1. *Lectures on Real Analysis by J. Yeh, World Scientific.*
2. *Theories of Integration by Douglas S. Kurtz & Charles W. Swartz, World Scientific.*

M4: Summability Methods-I **Full Marks: 50 Credit: 4**

Density, asymptotic density/ natural density, statistical convergence, statistical boundedness, statistical limit point, statistical cluster point, relation between statistical limit points set and statistical cluster points set, statistical limit superior and limit inferior, lacunary statistical convergence, strongly Cesaro summability, natural density of order α (where $0 < \alpha \leq 1$), statistical convergence of order α, λ-statistical convergence, strong (V, λ)-summability, $\alpha\beta$-statistical convergence, density by moduli, f-statistical convergence, f-statistical limit point, f-statistical cluster point, relation between f-statistical limit points set and f-statistical cluster points set, weighted statistical convergence, statistical convergence in probability.

Double natural density, statistical convergence of double sequences, statistical Cauchy sequence, statistical convergence of multiple sequences, relation between statistical convergence and strongly Cesaro summable sequences, statistical limit points of double sequences, double density of order (α, β) (where $0 < \alpha, \beta \leq 1$), statistical convergence of order (α, β), double density by moduli.

Ideal, filter, admissible ideal, maximal admissible ideal, I-convergence, I^*-convergence, I-boundedness, I-limit point, I-cluster point, I- $limsup\ x_n$, I- $liminf\ x_n$, I-Cauchy sequences, I-statistical and I-lacunary statistical convergence, I-statistical and I-lacunary statistical convergence of order α (where $0 < \alpha \leq 1$), I-statistically pre-Cauchy sequences, I and I^*-convergence of double sequences.

References:

1. Hemen Dutta and Billy E. Rhoades, Current Topics in Summability Theory and Applications, Springer (2016)
2. P.N. Natarajan, Classical Summability Theory, Springer (2017)
3. *Johan Boos and Peter Cass, Classical and Modern Methods in Summability, OXFORD University Press, (2000)*

M5: Topics in Analytic Number Theory **Full Marks: 50 Credit: 4**

Dirichlet series, Multiplication of Dirichlet series, Euler products.
Riemann zeta function, functional equation, zero free regions, analytic continuation.
Dirichlet L-function, Dirichlet's Theorem for primes in an arithmetic progression. Functional equation and Euler product for L-functions.
Chebyshev's ψ function, Chebyshev's θ function, Analytic proof of Prime Number Theorem.

References:

1. *T.M. Apostol, Introduction to Analytic number theory, Springer-Verlag (1976).*
2. *Harold Davenport, Multiplicative Number Theory, Springer.*
3. *S. Lang, Algebric Number Theory, Addison-Wesley, 1970.*
4. *M. Ram Murty, Problems in analytic number theory, springer.*
5. *M. Ram Murty, Michael Dewar and Hester Graves, Problems in the theory of modular forms, Hindustan Book Agency.*
6. *Neal Koblitz, Introduction to Elliptic curves and Modular forms, 2nd Ed., springer.*
7. *E. C. Titchmarsh, 2nd edition revised by D. R. Heath-Brown, The Theory of Riemann Zeta function, Oxford Science Publications.*
8. *K. Chandrasekharan, "Introduction to Analytic Number Theory", Springer-Verlag, 1968.*
9. *H. Iwaniec, E. Kowalski, "Analytic Number Theory", American Mathematical Society Colloquium Publications 53, American Mathematical Society, 2004.*

M6: Topological Indices of Graphs **Full Marks: 50 Credit: 4**

Topological graph, History of topological indices, Topological indices of graphs, Degree based and Distance based topological index, Wiener index, Zagreb index, Shannon Wiener index, F-index, Gutman index, Application of topological indices.

References:

1. *Topological indices and related descriptors in QSAR and QSPR, Gordon and Breach, Amsterdam, The Netherlands, 1999.*
2. *Mathematical concepts in organic chemistry (Springer- Verlag), Berlin, 1986.*

M7: Advanced Modern Algebra-I **Full Marks: 50 Credit: 4**

Categories, Functors, Natural Transformations, Universal Properties, Representable Functors, Yoneda's Lemma, Limits, Colimits, Adjoint Functors.
Group-Rings, Semisimple Rings and Modules.
Representation Theory of Finite Groups.

References:

1. *D.S. Dummit and R.M. Foote, Abstract Algebra (3e), John Wiley and Sons (Asian reprint).*
2. *J.J. Rotman, Advanced Modern Algebra, Springer (Indian reprint 2016).*
3. *N. Jacobson, Basic Algebra II, Dover Publication, 2018.*
4. *M.F. Atiyah and I.G. MacDonald, Introduction to commutative Algebra, CRC Press, 2019.*
5. *D. Eisenbud, Commutative Algebra, Springer, USA, 2004.*
6. *W.A. Adkins and S. H. Weintraub, Algebra, Springer-Verlag, 1999.*

M8: Finite and Boundary Element Methods **Full Marks: 50 Credit: 4**

Finite Element Method: Introduction to Finite Elements, FEM applied to discrete systems, Method of weighted residuals, FEM applied to one-dimensional linear static problems, FEM applied to two-dimensional linear static problems – scalar and vector field problems, introduction to dynamic problems.

Boundary Element Methods: Green's identity - Integral equation for the potential on the boundary, The Neumann and Dirichlet boundary conditions, Principle of boundary element method; conversion of basic weighted residue statement into boundary integral equation inverse statement, concept of fundamental solution; application to potential problem in two and three dimensions; type of boundary elements; Green's theorem - The Galerkin vector/"concentrated load" function, Somigliana's identity, The displacement and the traction boundary conditions, Direct vs. indirect formulations, Discretization of boundary into panels, Approximation of singularity distributions on the boundary, Galerkin vs. collocation approach, Matrix of influence coefficients, Evaluation of influence coefficients - The self-influence coefficient, Low-order (constant) vs. high-order (linear, quadratic) methods, Boundary shape discontinuities (corners), Applications.

References:

1. *C Pozrikidis, A Practical Guide to Boundary Element Methods with the Software Library BEMLIB, CHAPMAN & HALL/CRC, 2002.*
2. *J. T. Katsikadelis, Boundary Elements Theory and Applications, Elsevier, Oxford, 2002.*
3. *G. C. Hsiao, W. L. Wendland, Boundary Integral Equations, Springer-Verlag Berlin Heidelberg, 2008.*
4. *C Pozrikidis, Boundary Integral and Singularity Methods for Linearized Viscous Flow, Cambridge University Press, 1992.*

M9: Similarity Transformations and Perturbation Theory **Full Marks: 50 Credit: 4**

Prerequisite: Ordinary and Partial Differential Equations

General dimensional theory, Global similarity transformations, Transformation Groups, Infinitesimal Transformations, Invariant Functions, Prolongation and Invariance of Differential Equations (ODE and PDE), Invariant Solutions (Similarity Solutions). Parameter Perturbations, Coordinate Perturbations, Order Symbols and Gauge Functions, Asymptotic Expansions and Sequences, Straightforward expansions and sources of non-uniformity, Type change of a PDE, Method of Strained Coordinates, Method of matched and composite asymptotic expansions, Variation of parameters, Method of Multiple Scales.

References:

1. *Nayfeh, Ali Hasan, Introduction to perturbation techniques, John Wiley & Sons, 2011.*
2. *Murdock, James A, Perturbations: theory and methods, SIAM, 1999.*
3. *Bellman, Richard Ernest, Perturbation techniques in mathematics, engineering and physics, Courier Corporation, 2003.*
4. *Bluman, George W and Cole, Julian D, Similarity methods for differential equations, Springer Science & Business Media, 2012.*

M10: Advanced Numerical Techniques and MATLAB **Full Marks: 50 Credit: 4**

Prerequisite: Numerical Analysis, ODE, PDE

Revision of IVP and BVP, Single-step and Multi-step methods. System of first order ODE, higher order IVPs. Numerical solutions of BVP - Linear BVP, finite difference methods,shooting methods, stability, error and convergence analysis, nonlinear BVP, higher order BVP.

System of linear equations and eigenvalue problem: Operational counts for direct methods of solving system linear algebraic equations. Gaussian operational count for inversion of a matrix.Eigenvalue problem.General iterative method.Jacobi and Gauss.Seidel method.Relaxation method.Necessary and sufficient conditions for convergence. Speed of convergence. S.O.R. and S.U.R. methods. Gerschgorin’s circle theorem. Determination of eigenvalue by iterative methods, Ill conditioned system.

System of non-linear equations: Newton’s method. Existence of roots. Stability and convergence under variation of initial approximations. General iterative method for the system: $x = g(x)$ and its sufficient condition for convergence. The method of steepest descent.

Finite difference method: Grids, Finite-difference approximations to derivatives. Linear Transport Equation: Upwind, Lax-Wendroff and Lax-Friedrich schemes, VonNeumann stability analysis, CFL condition, Lax-Richtmyer equivalence theorem, Solution of partial differential equations by finite difference method. Partial difference quotients. Discretization error. Idea of convergence and stability. Explicit and Crank Nicolson implicit method of solution of one-dimensional heat conduction equation: convergence and stability. Standard and diagonal five point formula for solving Laplace and Poission equations. Explicit and Implicit method of solving Cauchy problem of one-dimensional wave equation, difference approximations in polar coordinates.

Finite Volume Methods: Basics and applications in solution of PDE.

Fundamentals of MATLAB, syntax, basic mathematical operations, Implementation of numerical schemes using MATLAB.

References:

1. *M. K. Jain, S.R.K. Iyenger and R.K. Jain, Computational Methods for Partial Differential Equations (Second edition), New Age International Publication (P) Ltd (2016).*
2. *K.W Mortons and D. F. Mayers, Numerical solution of partial differential equations (Second edition), Cambridge University press.*
3. *VitorianoRuas, Numerical Methods for Partial Differential Equations: An Introduction, Wiley (2016).*
4. *J. Kiusalaas, Numerical Methods in Engineering with MATLAB, Cambridge University Press, 2005.*
5. *M.K.Jain, Numerical solution of differential equations.*
6. *G.D.Smith, Numerical solution of partial differential equations.*
7. *S. R. Otto, James P. Denier, An Introduction To Programming And Numerical Methods In Matlab, Springer, 2005.*

M11: Modern Theory of Partial Differential Equations Full Marks: 50 Credit: 4

Prerequisites: Functional Analysis and Partial Differential Equations

Theory of distributions: supports, test functions, regular and singular distributions, generalized derivatives. Sobolev Spaces: definition and basic properties, approximation by smooth functions, dual spaces, trace and imbedding results. Elliptic Boundary Value Problems: abstract variational problems, Lax-Milgram Lemma, weak solutions and wellposedness with examples, regularityresult, maximum principles, eigenvalue problems. Semigroup Theory and Applications:exponential map, C0-semigroups, HilleYosida and Lummer-Phillips theorems, applications to heat and wave equations.

References:

1. *S. Kesavan, Topics in functional analysis and applications, (Wiley eastern, 1989).*
2. *L. C. Evans, Partial Differential Equations (second edition), AMS, Berkeley, 2010*
3. *M. Renardy, R. C. Rogers, An Introduction to Partial Differential Equations, Springer, 2004.*
4. *H. Brezis, Functional analysis, Sobolev spaces and Partial differential equations, Springer, 2011.*

M12: Advanced Trends in Continuum Mechanics **Full Marks: 50 Credit: 4**

Prerequisites: Continuum Mechanics

Nonlinear Elasticity: Preliminary Considerations, The Equilibrium Problem, Equilibrium Boundary Problems, Variational Formulation of Equilibrium, Isotropic Elastic Materials, Homogeneous Deformations, Homothetic Deformation, Extension of a Rectangular Block, Shear of a Rectangular Block, Universal Static Solutions, Constitutive Equations in Nonlinear Elasticity. Nondimensional Analysis of Equilibrium, Signorini's Perturbation Method for Mixed Problems, Signorini's Method for Traction Problems, Second-Order Hyperelasticity, Application of Signorini's Method.

Micropolar Elasticity: Preliminary Considerations, Kinematics of a Micropolar Continuum, Mechanical Balance Equations, Energy and Entropy, Elastic Micropolar Systems, The Objectivity Principle, Some Remarks on Boundary Value Problems, Asymmetric Elasticity.

Phase Equilibrium: Boundary Value Problems in Phase Equilibrium, Some Phenomenological Results of Changes in State, Equilibrium of Fluid Phases with a Planar and Spherical Interface Variational Formulation of Phase Equilibrium, Phase Equilibrium in Crystals, Wulff's Construction.

Introduction to Mixture Theory: Balance Laws, Classical Mixtures, Nonclassical Mixtures, Balance Equations of Binary Fluid Mixtures, Constitutive Equations, Phase Equilibrium and Gibbs' Principle.

Electromagnetism in Matter: Integral Balance Laws, Electromagnetic Fields in Rigid Bodies at Rest, Constitutive Equations for Isotropic Rigid Bodies, Approximate Constitutive Equations for Isotropic Bodies, Maxwell's Equations and the Principle of Relativity, Quasi-electrostatic and Quasi-magnetostatic Approximations, Balance Equations for Quasi-electrostatics, Isotropic and Anisotropic Constitutive Equations, Polarization Fields and the Equations of Quasi-electrostatics, More General Constitutive Equations.

Introduction to Magnetofluid Dynamics: An Evolution Equation for the Magnetic Field, Balance Equations in Magnetofluid Dynamics, Equivalent Form of the Balance Equations, Constitutive Equations, Ordinary Waves in Magnetofluid Dynamics, Alfven's Theorems, Laminar Motion Between Two Parallel Plates, Law of Iso-rotation.

Relativistic Continuous Systems: Lorentz Transformations, The Principle of Relativity, Minkowski Spacetime, Four-Dimensional Equation of Motion, Integral Balance Laws, The Momentum-Energy Tensor, Fermi and Fermi-Walker Transport, The Space Projector, Intrinsic Deformation Gradient, Relativistic Dissipation Inequality, Thermoelastic Materials in Relativity, Physical Meanings of Relative Quantities, Maxwell's Equation in Matter, Minkowski's Description, Amper's Model.

<u>References:</u>

1. *J. N. Reddy, An Introduction to Continuum Mechanics (Second edition), Cambridge University Press.*
2. *D. Rubin, E. Krempl, and W. Michael Lai, Introduction to Continuum Mechanics (Third edition), Pergamon press (1993).*
3. *Peter Chadwick, Continuum Mechanics: Concise Theory and Problems (Second Edition), Dover Publication Inc. (1999).*
4. *John W. Rudnicki, Fundamentals of Continuum Mechanics, John Wiley & Sons (2014).*
5. *A. J. M. Spencer, Continuum mechanics, Dover Publication Inc. (2004).*

M13: Advanced Fluid Mechanics **Full Marks: 50 Credit: 4**

Prerequisite: Ordinary and Partial differential equations, Vector and Tensor Calculus, Fluid Mechanics

Recapitulation: Concept of continuum and definition of a fluid. Body and surface forces, stress tensor.Scalar and vector fields, Eulerian and Lagrangian description of flow. Motion of fluid element; translation, rotation and vorticity; strain rate tensor, continuity equation, stream function and velocity potential.

Constitutive equations, derivation of Navier-Stokes equations. Exact solutions of Navier-Stokes equations: plane Poiseuille flow and Couette flow, Hagen-Poiseuille flow, flow between two concertic rotating cylinders, Stokes first and second problem, Hiemenz flow, flow near a rotating disk, flow in convergent-divergent channels. Slow viscous flow: Stokes and Oseen's approximation, theory of hydrodynamic lubrication. Thin-film equations. Boundary layer: derivation, exact solutions, Blasius, Falkner Skanseries solution and numerical solutions. Approximate methods.Momentum integral method.Two dimensional and axisymmetric jets.

Introduction to Hydrodynamic stability: linear stability of plane Poiseuille flow, Orr-Sommerfeld equation. Description of turbulent flow, velocity correlations, Reynolds stresses, Prandtl s Mixing Length Theory, Karman s velocity defect law, universal velocity distribution. Concepts of closure model, eddy viscosity models of turbulence- zero equation, one equation and two-equation models.

<u>References:</u>

1. *Frank. M. White, Fluid Mechanics, McGraw-Hill Higher Education, 8th edition.*
2. *I. Kohen and P. K. Kundu, Fluid Mechanics, Elsevier, 3rd edition.*
3. *Frank M. White, Viscous Fluid Flow, McGraw-Hill Higher Education, 3rd edition*
4. *George K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press.*
5. *S. K. Som, G. Biswas and S. Chakraborty, Introduction to Fluid Mechanics and Fluid Machines, Tata-McGraw-Hill, 3rd Edition.*
6. *S. W. Yuan, Foundation of Fluid Mechanics, Prentice-Hall, 1967.*

M14: Mathematical Methods **Full Marks: 50 Credit: 4**

Two-point boundary-value problems, Green's functions, Construction of Green's functions, Nonhomogeneous boundary conditions, Sturm-Liouville Systems, Eigen values and Eigen functions, Eigenfunction expansions and completeness.; Hypergeometric equation and functions, Properties of hypergeometric functions, Legendre equation and Legendre polynomial, Generating function for Legendre polynomial, Recurrence relations between Legendre polynomials, Rodrigueâs formula. Orthogonality of Legendre polynomial, Associated Legendre equation and Legendre function, Bessel equation and its solution, Bessel functions, Modified Bessel function, Generating function for Bessel function, Recurrence relations between Bessel functions, Orthogonality of Bessel functions.; Autonomous systems, Stability for Linear systems with constant coefficients, Linear plane autonomous systems, perturbed systems, Method of Lyapunov for nonlinear systems. Limit cycles of Poincare.; Coordinate transformations, Definition of Tensors, Summation convention, Kronecker Delta, Covariant, contravariant and mixed tensors. Fundamental operations with tensors, the line element and metric tensor, length of a vector, Christoffelâs symbols, the covariant derivative, tensor form of gradient, divergence and curl.Examples from continuum mechanics, elasticity, plasticity, fluids.

References:

1. *Arfken, George B and Weber, Hans J, Mathematical methods for physicists, AmericanAssociation of Physics Teachers, 1999.*
2. *Ken F. Riley, Mike P. Hobson, Stephen J. Bence, Mathematical Methods for Physics and Engineering, Cambridge, 2018.*
3. *Samuel D Lindenbaum, Mathematical Methods in Physics, World Scientific, 1996.*
4. *H. W. Wyld, Mathematical Methods for Physics, CRC Press, 2018.*

M15: Optimization Techniques **Full Marks: 50 Credit: 4**

Stochastic programming, chance constrained programming and two-stage programming, geometric programming, polynomial and signomial programming, dual based methods, primal based methods, Dynamic programming, single stage and multi-stage programming, forward and backward process, deterministic and probabilistic dynamic programming models, interior point methods, projective and scaling methods for linear programming.

References:

1. *C. B Gupta, Optimization Techniques in Operation Research, I.K. International Publishing House Pvt. Limited, 2008.*
2. *Xin-She Yang, Optimization Techniques and Applications with Examples, Wiley, 2018.*
3. *Cornelius T. Leondes, Optimization Techniques, Elsevier Science, 1998.*
4. *Chander Mohan, Kusum Deep, Optimization Techniques, New Age Science, 2009.*
5. *Sukanta Nayak, Fundamentals of Optimization Techniques with Algorithms, Elsevier Science, 2020.*

M16: Decision Theory **Full Marks: 50 Credit: 4**

Games and statistical games, statistical decision problem, decision function, risk function, prior and posterior distribution, Bayes risk and Bayes rules, least favourable prior, minimaxity, admissibility and complete classes, admissibility of Bayes rules, existence of minimal complete class and Bayes rules, the supporting and separating hyperplane theorems, essential completeness of the class of nonrandomized rules, minimax and complete class theorems, solving for minimax rules, essential completeness of class of rules based on sufficient statistics, continuity of risk functions, invariant decision problems, admissible and minimax invariant decision rules.

References:

1. *Martin Peterson, An Introduction to Decision Theory, Cambridge University Press, 2017.*
2. *Herman Chernoff, Lincoln E. Moses, Elementary Decision Theory, Dover Publications, 2012.*
3. *Douglas John White, Decision Theory, Transaction Publishers, 1969.*
4. *Giovanni Parmigiani, Lurdes Inoue, Decision Theory Principles and Approaches, Wiley, 2009.*

M17: Differential Geometry of Manifolds **Full Marks: 50 Credit: 4**

Differentiable manifolds: Definition and Examples of Topological manifolds, smooth maps and diffeomorphisms , Definition and examples of differentiable manifolds, derivatives of smooth maps, local expression for the differential, curves in a manifold, immersion and submersion, rank, critical and regular points, submanifolds and regular submanifolds. Tangent and Cotangent spaces, Tangential maps, Vector Fields on smooth manifolds, Lie Brackets and its properties, Integral Curves and Flows, f-related vector fields, 1-parameter group of transformations, Differential forms, local expression for a k-form, pull back of a k-form, wedge product, Exterior algebra and Exterior derivatives.

Riemannian manifolds: Affine connections, Riemannian and semi-Riemannian metrics, Riemannian connection, Riemann curvature tensor, Ricci tensor, Scalar curvature, Sectional Curvature, Semi-symmetric and quarter symmetric metric connections on Riemannian manifolds, Einstein manifolds and its generalizations, Manifolds of constant curvature and its generalizations, Some transformations (e.g., Conformal transformation, Projective transformation, Concircular transformation, Conharmonic transformation) on Riemannian manifolds, Locally symmetric Riemannian manifolds due to Cartan and its generalizations, Product manifolds, Warped product manifolds.

Submanifolds: Embedded Submanifolds, Immersed Submanifolds, Hypersurfaces of Riemannian manifolds, Induced connection and second fundamental form, Gauss and Weingarten formulae, Equations of Gauss, Codazzi and Ricci, Mean curvature, Totally geodesic and totally umbilical submanifolds, Minimal submanifolds.

References:

1. *W. M. Boothby; An Introduction to Differentiable Manifolds and Riemanian Geometry; Academic Press, Revised, 2003.*
2. *L. Conlon; Differentiable Manifolds, A First Course; Birkhauser (Second Edition), 2008.*
3. *W. D. Curtis and F. R. Miller; Differential Manifolds and Theoretical Physics; Academic Press, 1985.*
4. *S. Helgason; Differential Geometry, Lie Groups and Symmetric Spaces; Academic Press, 1978.*
5. *N.J. Hicks; Notes on Differential Geometry; Notes.*
6. *Kobayashi & Nomizu; Foundations of Differential Geometry, Vol-I; Interscience Publishers, 1963.*
7. *S. Kumaresan; A course in Differential Geometry and Lie-groups; Hindustan Book Agency.*
8. *S. Lang; Differential and Riemannian manifolds; Springer-Verlag, 1995.*
9. *John M. Lee; Introduction to smooth manifolds; Springer.*
10. *M. Spivak; A Comprehensive Introduction to Differential Geometry, volumes 1 and 2; Publish or Perish, 1979*
11. *K. Yano and M. Kon; Structure on Manifolds; World Scientific, 1984*
12. *T. J. Willmore, Riemannian Geometry, Oxford University Press, 1997.*
13. *K. Yano and M. Kon, Structure on Manifold, World Scientific Publication, Singapore, 1984.*
14. *J. M. Lee, Riemannian Manifolds, An Introduction to Curvature, Springer-Verlag, 2005.*
15. *B. Y. Chen, Geometry of Submanifolds, Marcel Dekker. Inc., New York, 1973.*

M18: Mathematical Theory of Elasticity **Full Marks: 50 Credit: 4**

Prerequisite: concept of tensor

Analysis of strain: Introduction, affine transformation, infinitesimal affine deformation, a geometrical interpretation of the components of strain, strain quadric of Cauchy, principal strains, invariants, general infinitesimal deformation, equations of compatibility, finite deformations.

Analysis of stress: Body and surface forces, stress tensor, equations of equilibrium, transformations of coordinates, stress quadric of Cauchy, maximum normal and shear stresses.

Stress-strain relations: Hooke's law, Generalized Hooke's law, Homogeneous and isotropic bodies, elastic moduli of isotropic bodies, elastic moduli of isotropic bodies, equilibrium equations for an isotropic elastic solid, dynamical equations of an isotropic elastic solid, the strain-energy function and its connection with Hooke's law, uniqueness of solution of the boundary-value problems of Elasticity, Saint-Venant's principle.

Waves in elastic media: Body waves of dilatation and distortion. Surface waves-Rayleigh and love waves.

Thermoelasticity: Introduction to Thermoelasticity, Basic equations of thermoelasticity, hyperbolic thermoelasticity.

References:

1. *I. S. Sokolnikoff, Mathematical theory of elasticity.*
2. *A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity.*

Semester-II

N1: General Theory of Integration **Full Marks: 50 Credit: 4**

Tagged Gauge Partitions. Definitions, Cousins Theorem, Right-left Procedure, Straddle Lemma, Application in continuity, Intrinsic Power.
Henstock–Kurzweil Integral. Definition and basic properties. Fundamental Theorem, Saks-Henstock Lemma, Inclusion of the Lebesgue integral. Squeez Theorem, Vitali-Covering Theorem, Differentiation Theorem, Characterization Theorem.

Reference:

1. *A Modern Theory of Integration, R. G. Bartle, AMS*
2. *Theories of Integration, Douglas S. Kurtz & Charles W. Swartz, World Scientific.*
3. *Lanzhou Lectures on Henstock Integration, Lee Peng Yee, World Scintific.*
4. *The Riemann, Lebesgue and General Riemann Integrals, A.G. Das, Narosa.*
5. *The general Theory of integration, R. Henstock, Clarendon Press.*

N2: Value Distribution Theory of Meromorphic Functions **Full Marks: 50 Credit: 4**

Uniqueness of entire and meromorphic functions, Nevanlinna's five value theorem, Distribution of the values of meromorphic functions and their derivatives, Milloux theorem, Exceptional values of meromorphic functions and their derivatives, Multiple value and uniqueness.

References:

1. *W. K. Hayman: Meromorphic functions, The claredon Press, Oxford, 1964.*
2. *C. C. Yang and H. X. Yi: Uniqueness Theory of Meromorphic Functions, Science Press, Beijing, 2003.*
3. *G. Valiron: Lectures on the general theory of integral functions, Chelsea Publishing Company, 1949.*

N3: Summability Methods-II **Full Marks: 50 Credit: 4**

Rough convergence, dependence on roughness degree, rough Cauchy sequences, rough limit set, core of a sequence, Chebyshev centers, rough statistical convergence, rough statistical cluster point, rough I-convergence, rough I-convergence of double sequences, rough weighted statistical convergence, rough weighted I-convergence, rough weighted I-cluster point, rough weighted I-lacunary statistical convergence, rough Wijsman convergence, asymptotic cones, characterization of uniform rotundity in every direction in terms of rough convergence, rough convergence in probability.

References:

1. Hemen Dutta and Billy E. Rhoades, Current Topics in Summability Theory and Applications, Springer (2016)
2. P.N. Natarajan, Classical Summability Theory, Springer (2017)
3. *Johan Boos and Peter Cass, Classical and Modern Methods in Summability, OXFORD University Press, (2000)*
4. *Shyamal Debnath and Debjani Rakshit, Rough Convergence in Metric Spaces, Trends in Mathematics, pp 449-454*

N4: Advanced Modern Algebra-II **Full Marks: 50 Credit: 4**

Semigroups. Rees Congruences. Green's Equivalence Relations. Regular Semigroups. Completely Regular Semigroups. Semilattices of Groups. Clifford Semigroups. Orthodox Semigroups.
Semirings. Quotient Semirings. Additively Regular Semirings. Euclidean Semirings. Completely Regular Semirings.
Lattice. Distributive Lattice. Modular Lattices. Characterization and Representation Theorems. Generalized Boolean Algebra.

References:

1. *A. H. Clifford and G. B. Preston, The algebraic theory of semigroups, AMS, Math. Surveys No. 7, Vol I, 1961, Vol II, 1967.*
2. *J. M. Howie, An introduction to semigroup theory, Academic Press, London, 1976.*
3. *J. S. Golan, Semiring and their Applications, Kluwer Academic Pub., Boston, 1999.*
4. *U. Hebisch and H. J. Weinert, Semirings, World Scientific, Singapore, 1993.*
5. *G. Birkhoff, Lattice Theory, AMS, Colloquium Pub. Vol. 25, New York, 1948.*

N5: Modular Forms **Full Marks: 50 Credit: 4**

The full modular group and its congruence subgroups, Hecke subgroups, Eisenstein series.
Modular forms for $SL_2(\mathbb{Z})$, Modular forms for congruence subgroups, differential operators. Structure of the ring of modular forms.
Hecke operators and Euler product for modular forms. The L-function of a modular form, functional equations. Theta functions, transformation formula, sums of four squares.

References:

1. *S. Lang, "Introduction to Modular Forms", Springer-Verlag, 1995.*
2. *J.-P. Serre, "A Course in Arithmetic", Graduate Texts in Mathematics 7, Springer-Verlag, 1973.*
3. *N. Koblitz, "Introduction to Elliptic Curves and Modular Forms", Graduate Texts in Mathematics 97, Springer-Verlag, 1993.*
4. *J. H. Bruinier, G. van der Geer, G. Harder, D. Zagier, "The 1-2-3 of Modular Forms", Universitext, Springer-Verlag, 2008.*
5. *F. Diamond, J. Shurman, "A First Course in Modular Forms", Graduate Texts in Mathematics 228, Springer-Verlag, 2005.*
6. *M Ram Murty, Problems in the theory of modular forms, Hindustan book agency, 2015.*

N6: Complex Manifolds **Full Marks: 50 Credit: 4**

Riemannian manifolds, Affine Connections (Koszul), Torsion and Curvature tensor field on Affine Connection, Covariant Differential. Almost Complex Manifolds: Introduction, algebraic Preliminaries, Nijenhuis tensor, Eigen values of the complex structure, Existence theorem and Integrability condition of an almost complex structure, Contravariant and covariant almost analytic vector field, Almost Hermite Manifolds, curvature tensor, Holomorphic sectional curvature, Linear connection in an almost Hermite manifold, Kähler Manifold: Kähler Manifolds, Holomorphic sectional curvature, Bochner curvature tensor, Affine connection in Kähler manifolds, Conformally flat Kähler manifolds, Projective correspondence between two Kӓhler manifolds, Nearly Kӓhler Manifolds, Para Kӓhler Manifolds, conformal flatness of para Kӓhler manifolds.

Submanifolds of Kӓhler manifolds: Kӓehlerian submanifolds, Anti-invariant submanifolds of Kӓehlerian manifolds, CR-submanifolds of Kӓehlenian manifolds.

References:

1. *R.S.Mishra; Structures on a Differentiable Manifold and Their Applications; Chandrama Prakashan, Allahabad, 1984.*
2. *K.Yano and M.Kon; Structures on Manifolds; World Scientific, 1984.*
3. *S.S. Chern; Complex Manifolds Without Potential Theory; New York, Springer-Verlag, 1979.*
4. *P. Griffiths and J. Harris; Principles of Algebraic Geometry; New York, John Wiley & Sons, 1978.*
5. *R. C. Gunning; Lectures on Riemann Surfaces; Princeton, Princeton University Press, 1966*
6. *S. Kobayashi and K. Nomizu; Foundations of Differential Geometry; New York, Interscience Publishers, 1969*
7. *R. O. Wells, Jr.; Differential Analysis on Complex Manifolds; New York, Springer-Verlag, 1980.*
8. *F. Zheng; Complex Differential Geometry; Providence, American Mathematical Society, 2000.*

N7: Contact Manifolds **Full Marks: 50 Credit: 4**

Contact manifold, contact metric manifold, almost contact manifold, Torsion tensor of almost contact metric manifold, Killing vector field, properties of ϕ, the tensor field h, some curvature properties of contact metric manifold. K-contact Manifolds , Characterizations of K-contact manifolds, some curvature properties of K-contact manifolds, sectional curvature of K-contact manifolds, Locally symmetric and Ricci symmetric K-contact manifolds, semi-symmetric and Ricci - semisymmetric K-contact manifolds.

Sasakian manifolds : Introduction, some curvature properties, ϕ sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor, N(k)-Contact Metric Manifolds : k-nullity distribution, η-Einstein N(k)-Contact Metric manifolds, Conformally flat N(k)-contact metric manifolds, some curvature properties, Almost para-contact structure, Torsion tensor fields, Examples of paracontact manifolds, P-Sasakian manifolds.

Submanifolds of Sasakian Manifolds: Invariant submanifolds of Sasakian manifolds, Anti-invariant submanifolds tangent to the structure vector field of Sasakian manifolds, Anti-invariant submanifolds normal to the structure vector field of Sasakian manifolds.

References:

1. *R.S.Mishra; Structure on a Differentiable manifold and their Applications; Chandrama Prakashani, Allahabad, 1984.*
2. *K.Yano and M.Kon; Structures on Manifolds; World Scientific, 1984.*
3. *Blair, D. E.; Contact manifolds in Riemannian geometry; Lecture note in Math., 509, Springer-Verlag, Berlin-New York, 1976.*
4. *Blair, D. E.; Riemannian geometry of contact and symplectic manifolds; Progress in Math., 203, Birkhauser Boston, Inc., Boston, 2002.*
5. *John M. Lee, Riemannian manifolds: An introduction to curvature, Springer-Verlag, 1997.*

N8: General Theory of Relativity and Cosmology **Full Marks: 50 Credit: 4**

Minkowski space-time: Past and future Cauchy development, Cauchy surface. DeSitter and anti-de Sitter space-times. Robertson-Walker spaces. Spatially homogeneous space-time models. The Schwarzschild and Reissner – Nordstrom solutions. Kruskal diagram. Causal structure. Orientability. Causal curves. Causality conditions. Cauchy developments. Global hyperbolicity. The existence of Geodesics. The Causal boundary of space-time. Asymptotically simple spaces.

What is cosmology? Homogeneity and isotropy of the universe. The Weyl Postulate. The cosmological principle. General relativistic cosmological models. Cosmological observations. The Olbers Paradox. The Friedman Cosmological Models (dust and radiation models). Cosmologies with a non-zero λ. Hubble's Law, the age of the4 Universe. Gravitational red shift and Cosmological redshift.

The spherically symmetric space-time: Schwarzschild solution. Patrick orbits in the Schwarzschild space-time. Newtonian approximation. Photon orbits. Birkhoffs theorem. Equilibrium of Massive spherical objects. The Schwarzschild Interior solution. The interior structure of the star. Realistic stars and gravitational collapse. White dwarfs, Neutron stars.

Gravitational collapse of a homogeneous dust ball. Schwarzschild black hole. Simple idea of black hole physics.

References:

1. *General Relativity and Cosmololgy – J.V. Narlikar.*
2. *A first course in general relativity – B.F. Schultz.*
3. *Introduction to cosmology - J.V. Narlikar.*
4. *An Introduction to Mathematical Cosmology – J.N. Islam (Camb. Univ. Press).*
5. *Gravitation and Cosmology – S. Weinberg (J. Wiley and Sons.)*
6. *General Relativity, Astrophysics and Cosmology – Raychaudhuri, Banerji and Banerjee (Springer-Verlag).*
7. *Introduction to Cosmology – M. Ross (J. Wiley and Sons).*

N9: Advanced Quantum Mechanics **Full Marks: 50 Credit: 4**

Fundamental ideas of quantum mechanics: Nature of the electromagnetic radiation; Wave-particle duality - double-slit experiment, quantum unification of the two aspects of light, matter waves; Wave functions and Schrodinger equation; Quantum description of particle - wave packet, uncertainty relation.

Mathematical formalism of quantum mechanics: Wave function space – bases, representation; State space – bases, representation; Observables – R and P observables; Postulates of quantum mechanics.

Physical interpretation of the postulates: Statistical interpretation – expectation values, Ehrenfest theorem, uncertainty principle; Physical implications of the Schrodinger equation - evolution of physical systems, superposition principle, conservation of probability, equation of continuity; Solution of the Schrodinger equation – time evolution operator, stationary state, time-independent Schrodinger equation; Equations of motion – Schrodinger picture, Heisenberg picture, interaction picture.

Theory of harmonic oscillator: Matrix formulation – creation and annihilation operators; Energy values; Matrix representation in n basis; Representation in the coordinate basis; Planck's law; Oscillator in higher dimensions.

Symmetry and conservation laws: Symmetry transformations – basic concepts, examples; Translation in space; Translation of time; Rotation in space; Space inversion; Time reversal.

Angular momentum: Orbital angular momentum - eigen values and eigen functions of L_2 and L_z; Angular momentum operators $\vec{J}$– commutation relations, eigen values and eigen functions; Representations of the angular momentum operators.

Spin: Idea of spin – Bosons, Fermions; Spin one-half – eigen functions, Pauli matrices; Total Hilbert space for spin-half particles; Addition of angular momenta; Clebsch-Gordan coefficients–computation, recursion relations, construction procedure; Identical particles - symmetrisation postulate, Pauli exclusion principle, normalization of states.

One-electron atom: Schrodinger equation; Energy levels, Eigen functions and bound states, Expectation values and virial theorem; Solution in parabolic coordinates; Special hydrogenic atom (brief description) – positronium, muonium, antihydrogen, Rydberg atoms.

Time-independent perturbation theory: Basic concepts; Derivation – up to the second order correction to the energy values and wave functions; Applications - anharmonic oscillator; normal helium atom, ground state of hydrogen and Stark effect. Variational method: Rayleigh-Ritz variational principle; Applications – one dimensional harmonic oscillator, hydrogen atom, helium atom.

Relativistic quantum mechanics: Klein-Gordon equation – plane wave solution, interpretation of K-G equation; Dirac equation – covariant form, charged particle in electromagnetic field, equation of continuity, plane wave solution; Dirac hole theory; Spin of the Dirac particle.

References:

1. B. H. Bransden and C. J. Joachain, Quantum Mechanics, Prentics Hall (2005); Physics of Atoms and Molecules, Pearson Education, 2007.
2. A. Das, Lectures on Quantum Mechanics, Hindusthan Book Agency, New Delhi, 2003.
3. C. Cohen-Tannoudji, B. Diu, and F. Laloe, Quantum Mechanics Vol. 1, Wiley- Interscience publication, 1977.
4. D. J. Griffiths, Introduction to Quantum Mechanics, Pearson Prentics Hall, Upper Saddle River, NJ, 2005.
5. L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, 1968.

N10: Boundary Layer Theory and Turbulence **Full Marks: 50 Credit: 4**

Brief history and development of Fluid Mechanics, Fluid properties; Newtonian and non-Newtonian fluids, Different models of non-Newtonian fluids, Nanofluids and its different models, Integral laws for conservation of mass, momentum, angular momentum and energy; Constitutive laws, Differential forms of mass conservation equation, Navier-Stokes Equations; Differential form of Energy equation. Scaling and dimensional analysis, Dynamic Similarity, Laminar and Turbulent flows, Pipe flow, Open channel flow, Boundary layer theory, similarity solutions, high Re flows, creeping flows, steady and unsteady flows, heat transfer coefficients, molecular diffusion in fluids, mass transfer coefficients, Statistical interpretation of turbulence and its analysis, Spectral analysis and Kolmogorov theory.
Fundamentals of finite difference methods – explicit and implicit schemes; numerical stability and numerical solutions to non-linear ordinary and partial differential equations.

References:

1. *G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 2005.*
2. *S.W. Yuan, Foundations of Fluid Mechanics, Prentice – Hall International, 1970.*
3. *H. Schlichting, Boundary Layer Theory, Springer, 2003.*
4. *P.K. Kundu and I.M. Cohen, Fluid Mechanics, 3rd Ed., Academic Press, 2004.*
5. *F.S. Sherman, Viscous Flow, McGraw Hill International, 1990.*
6. *K. Muralidhar and G. Biswas, Advanced Engineering Fluid Mechanics, 2nd Ed., Alpha Science, 2005.*
7. *F.M. White, Viscous Fluid Flow, McGraw Hill International, 1991.*

N11: Chaos Theory and Fractals **Full Marks: 50 Credit: 4**

Prerequisite: Ideas on dynamical systems both continuous and discrete systems, fixed points, periodic points, periodic cycles and their stabilities, bifurcations theory and some important maps.

Topological conjugacy, properties of conjugacy, semi-conjugacy relations.

Mathematical theory of chaos: Sensitive dependence on initial condition (SDIC), topological transitivity and mixing, definition of chaotic map. Examples of chaotic maps, ergodic map and ergodic theorem, dynamics of logistic map for $r \geq 4$, symbolic dynamics.

Quantifying chaos: Universal sequence, Feigenbaum number, renormalization group theory and super-stable cycle, Lyapunov exponent and invariant measure.

Sharkovskii's theorem, Li and Yorke theorem, Poincare map, circle map and Smale Horseshoe map. Routes of chaos, universality in chaos.

Fractals: Self-similarity and scaling, self-similar fractals, constructions of self-similar fractals, dimensions of fractals, strange attractors, fractal basin boundary. Applications to fractals in chaotic dynamics and biological systems.

References:

1. *S. Sternberg, Dynamical systems, Diver, 2010.*
2. *R. L. Devaney, An Introduction to Chaotic Dynamical Systems, CRC Press, 2003.*
3. *G.C. Layek, An Introduction to Dynamical Systems and Chaos, Springer, 2015.*
4. *Edward Ott, Chaos in Dynamical Systems, 2nd Ed, CUP, 2002.*
5. *K.T. Alligood, T.D. Sauer and J.A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer, 1997.*
6. *K. Falconer, Fractal Geometry, Mathematical Foundation and Applications, Wiley, New York, 1990.*
7. *S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, 2nd Ed, Springer, 2003.*
8. *B. Mandelbrot, The Frcatal Geometry of Nature, Freeman & Company, 1982.*

N12: Non-Linear Partial Differential Equations **Full Marks: 50 Credit: 4**

First-order non-linear PDE: First-order nonlinear equations and their applications, the generalized method of characteristics, complete integrals of certain special nonlinear equations, the Hamilton-Jacobi equation and its applications.

Second-order non-linear PDE: Non-linear model equations and variational principles, basic concepts and definitions, some nonlinear model equations, variational principles and the Euler-Lagrange equations, the variational principle for nonlinear Klein-Gordon equations.

Conservation laws: Conservation laws and shock waves, conservation laws, discontinuous solutions and shock waves, weak or generalized solutions.

References:

1. *Lokenath and Debnath, Nonlinear Partial Differential Equations for Scientists and Engineers, Springer, 2nd Ed., 2005.*
2. *J. D. Logan, An Introduction to Non-linear Partial Differential Equations, John Wiley and Sons, 2010.*
3. *L. C. Evans, Partial Differential Equations, Vol.19, American Mathematical Society, 2nd Ed., 2010.*

N13: Fuzzy Sets and Applications **Full Marks: 50 Credit: 4**

Fuzzy sets: Basic definitions, level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, Cartesian products, algebraic products bounded sum and difference.

Extension principle and application: Zadeh extension principle, image and inverse image of fuzzy sets, fuzzy numbers, elements of fuzzy arithmetic.

Fuzzy relations: Fuzzy relations on fuzzy sets, union and intersection of fuzzy relation, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relation.

Fuzzy decision: Fuzzy linear programming problems: Symmetric fuzzy linear programming problem, fuzzy linear programming with crisp objective function, fuzzy graph.

Fuzzy logic: An overview of classic logic, its connectives, tautologies, contradiction fuzzy logic, fuzzy quantities, logical connectives for fuzzy logic, applications to control theory.

References:

1. *G. J. Klirand and B. Yuan, Fuzzy Sets and Fuzzy Logic Theory and Applications, Prentice Hall of India 1995.*
2. *T. J. Ross, Fuzzy Logic with Engineering Applications, McGraw Hill, International Editions, 2010.*
3. *J. J. Buckley and E. Eslami, An Introduction to Fuzzy Logic and Fuzzy Sets, Springer-Verlag Heidelberg, 2002.*
4. *G. Chen and T. T. Pham, Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems, CRC Press LLC, N.W. Florida, 2000.*

N14: Mathematical Logic **Full Marks: 50 Credit: 4**

Programming Prolog - facts, simple queries, complex queries, rules, arithmetic operators, recursion, unification, lists, cut. Propositional logic - syntax, semantics, laws of deduction, normal forms, resolution, theorem proving, validity, soundness and completeness. First order logic - conversion of common sense sentences into the language of first order logic, universal and existential quantifiers, syntax, terms of predicate, model theoretic semantics, Harbrand universe, normal form, unification, proof theory, mechanical theorem proving, incompleteness.

References:

1. *Richard E. Hodel, An Introduction to Mathematical Logic, PWS Publishing Company, 1995.*
2. *Elliott Mendelson, Introduction to Mathematical Logic, CRC Press, 2015.*
3. *Michał Walicki, Introduction to Mathematical Logic, World Scientific Publishing Company, 2011.*

N15: Multivariate Analysis **Full Marks: 50 Credit: 4**

Multiple linear regression, problems of multicollinearity, heteroscedasticity and their remedies, autocorrelation and its remedial measures, polynomial regression and the method of orthogonal polynomials, nonlinear regression - some specific models and solutions. Testing general linear hypothesis - testing equality of means of several normal distributions with common covariance matrix, MANOVA, testing independence of a set of variates, testing hypotheses of equality of covariance matrices. Confounding in factorial experiments, fractional replications, split-plot designs. BIBD and PBIBD. Factor analysis.

References:

1. *Parimal Mukhopadhyay, Multivariate Statistical Analysis, World Scientific, 2009.*
2. *Neil H. Timm, Applied Multivariate Analysis, Springer New York, 2007.*
3. *Wolfgang Karl Härdle, Léopold Simar, Applied Multivariate Statistical Analysis, Springer Berlin Heidelberg, 2015.*
4. *Bernard Flury, A First Course in Multivariate Statistics, Springer, 1997.*
5. *W. J. Krzanowski, Wojtek Krzanowski, Principles of multivariate analysis: a user's perspective, OUP Oxford, 2000.*

N16: Numerical Optimization **Full Marks: 50 Credit: 4**

Basics of numerical optimization algorithms and their convergence properties.

Derivative free algorithms and their convergence analysis: (I) Single Dimension-Golden section and Fibonacci search methods (II) Multidimension- Coordinate search method, Method of Hookes and Jeeves, Method of Rosenberg.

Derivative based algorithms and their convergence analysis: Steepest descent method, Newton method, Conjugate gradient methods, Quasi Newton methods, Trust region methods, Penalty, Barrier, and Augmented Lagrangian methods, Sequential Quadratic Programming Technique.

References:

1. *Joseph-Frédéric Bonnans, Jean Charles Gilbert, Claude Lemarechal, Claudia A. Sagastizábal, Numerical Optimization Theoretical and Practical Aspects, Springer Berlin Heidelberg, 2013.*
2. *Jorge Nocedal, Stephen Wright, Numerical Optimization, Springer New York, 2006.*
3. *Suresh Chandra, Jayadeva, Aparna Mehra, Numerical Optimization with Applications, Alpha Science International, 2009.*
4. *Éric Walter, Numerical Methods and Optimization: A Consumer Guide, Springer International Publishing, 2014.*

N17: Lie Groups and Lie Algebras **Full Marks: 50 Credit: 4**

Transformation groups, orthogonal groups in 3- space, Euclidean group E(2), symmetry and discrete groups of E(3), group representation, reducible and irreducible representations, group characters, exponential of a matrix, local Lie groups, classical groups, Lie algebras, solvable and nilpotent Lie algebras, semi simple Lie algebras, classical Lie algebras, exponential map of a Lie algebra.

References:

1. *Arthur A. Sagle, Ralph Walde, Introduction to Lie Groups and Lie Algebras, Elsevier Science, 1973.*
2. *Robert Gilmore, Lie Groups, Lie Algebras, and Some of Their Applications, Dover Publications, 2012.*
3. *Brian Hall, Brian C. Hall, Lie Groups, Lie Algebras, and Representations: An Elementary Introduction, Springer, 2003.*
4. *Alexander Kirillov, Jr, Kirillov, An Introduction to Lie Groups and Lie Algebras, Cambridge University Press, 2008.*

N18: Queuing Theory **Full Marks: 50 Credit: 4**

Probability and random variable, discrete and continuous univariate and multivariate distributions, moments, law of large numbers and central limit theorem (without proof), Poisson process, birth and death process, infinite and finite queueing models M/M/1, M/M/C, M/G/1, M/M/1/N, M/Ek/1, Ek/M/1, M/G/1/N, GI/M/1, priority queueing models, network of queues, finite processor sharing models, central server model of multiprogramming, performance evaluation of systems using queueing models. Concepts of bottleneck and system saturation point. Introduction to discrete time queues and its applications.

References:

1. *P. Kandasamy, K. Gunavathy, Probability and Queuing Theory, S. Chand Limited, 2008.*
2. *U. Narayan Bhat, An Introduction to Queueing Theory Modeling and Analysis in Applications, Birkhäuser Boston, 2015.*
3. *Donald Gross, John F. Shortle, James M. Thompson, Carl M. Harris, Fundamentals of Queueing Theory, Wiley, 2011.*
4. *R.H.Chitale, Probability And Queuing Theory, Technical Publications, 2008.*

N19: Data Structure and Algorithm **Full Marks: 50 Credit: 4**

Stack and queues, linked list, Direct address tables, Indexing, hash tables, open addressing, trees, Binary search tree, height balanced tree, Red-black tree, B-tree. Basic concepts of algorithms, Complexity, Asymptotic notations, Trees: Binary tree, Binary Search Tree, Tree traversals. Heap as data structure. Basic sorting algorithms: selection sort, insertion sort. Greedy algorithms: Coin change problem, activity selection, Minimum Spanning Tree, Single source shortest path, knapsack problem. Divide â and â Conquer technique: Merge sort, quick sort. Solving Recurrence relations. Dynamic programming: matrix chain multiplication, all pair shortest path algorithm. Graph algorithms: Warshall’s algorithm, Depth First Search, Breadth First Search. Branch and Bound technique, Backtracking. NP completeness.

References:

1. *N. A. Deshpande S. S. Sane, Data Structures and Algorithms, Technical Publications, 2006.*
2. *Maria S. Rukadikar, Data Structures and Algorithms for Beginners, Arizona Business Alliance LLC, 2014.*
3. *Mayank Patel, Data Structure and Algorithm With C, Educreation Publishing, 2018.*
4. *A. A. Puntambekar, Advanced Data Structures and Algorithms, Technical Publications, 2008.*
5. *A. A. Puntambekar, Data Structures and Algorithms, Technical Publications, 2009.*
6. *Aho Alfred V., Aho, Data Structures and Algorithms, Pearson Education, 1983.*